Birla Institute of Technology & Science, Pilani

Comprehensive Exam **I Semester 2022-23**

Course Name: Numerical Analysis (MATH F313) **Date: 26-12-2022**

Max. Time: 90 Min. **Max. Marks: 46**

Note: 1. The comprehensive question paper consists of two parts, Closed Book and Open Book. Attempt questions of Closed Book and Open Book in two separate answer sheets provided.

2. Each subpart of a particular question should be in continuation.

3. Write on top right corner of two answer books Closed Book and Open Book**.**

4. Use four significant digits with rounding in all calculations where not specified.

(Closed Book)

1. The velocity V of a particle at a distance S from a point on its linear path is given in the following table:

S(m):	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
V(m/sec):	16	19	21	22	20	17	13	11	9

Estimate the time taken by the particle to traverse the distance of 20 meters, using composite Simpson's 1/3 rd rule. [6]

2. Derive three point Gauss-Legendre formula **with error term** and then using the derived formula without error term, evaluate $\int_0^{\pi/2} \sin x\, dx$. [12]

3. Using Divided differences (fitting cubic polynomial), derive the 4[th] order Adams-Bashforth-Moultan **predictor formula** without error term to find $y(x_{n+1})$as a solution of $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ (with spacing h). [8]

4. The equation $xe^{1-x} = 1$ has a root at $x = 1$. Starting with $x_0 = 0$, find the above root by a suitable method of quadratic convergence (Do four iterations). Also, define the order of convergence. **[6]**

5. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5
Distance	0	225	383
Speed	75	77	80

Using polynomial approximation of all the data points, predict the position and speed of the car when time t = 2 seconds (use five digits after decimal place with rounding) [**10]**

6. Using Power method, find the dominant Eigen value and the corresponding eigenvector for the following matrix [take $x^0 = (1,1,1)^T$]. (Perform four iterations only)

$$\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$ **[4]**

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Comprehensive Exam (Open Book) **I Semester 2022-23**

Course Name: Numerical Analysis (MATH F313) **Date: 26-12-2022**

Max. Time: 90 Min. **Max. Marks: 44**

Note: 1. Use four significant digits with rounding in all calculations where not specified.

(Open Book)

1. Apply the second order finite difference method to the following boundary value problem
$(1+x^2)y''+2xy'-y=1+x^2$, with $2y'(-1)=3y(-1),\ y'(1)=4y(1)$,
to obtain a system of linear equations in terms of y(-1), y(0), and y(1) by taking *h=1*. Hence, perform one iteration of Gauss-Seidel method to solve the resulting system of equations which obtained from FDM with initial approximation as $[1, 1, 1]^T$. [12]
2. A curve $\boldsymbol{y=f(x)}$ whose slope at any point is equal to $\boldsymbol{-y^2}$ and passes through the point $\boldsymbol{(0,1)}$. Find the interpolating polynomial $\boldsymbol{P_n(x)}$ which interpolates the above function $\boldsymbol{f(x)}$ at $\boldsymbol{x=0,1}$ based on the values of $\boldsymbol{f(0), f(1), f'(1)}$ & $\boldsymbol{f''(1)}$. (Do not write entries of the difference table and coefficients of the polynomial in floating point notations, i.e. keep them as fractions only). **[6]**
3. Use Collocation method with trial function as a fourth degree polynomial of the form $c_0+c_1x+c_2x^2+c_3x^3+c_4x^4$ to obtain a system of linear equation for the following boundary value problem
$$\boldsymbol{y''-y'=2x,\ \text{with } y(0)=-1, y(1)=-1,}$$
so that residual at points 0.2, 0.5 and 0.7 become zero. Hence, apply Gauss-Elimination method with scaled partial pivoting to solve the resulting system of equations which obtained from Collocation method. [14]
4. For the following initial value problem
$$\boldsymbol{\frac{d}{dx}[y''+x(y-\tfrac{x}{2})]-3y=0,}$$
$$\boldsymbol{y(0)=0,\ y'(0)=1, y''(0)=0,}$$
find the approximate value of $y(0.2)$, $y'(0.2)$, $y''(0.2)$ taking h=0.2 using Runge-Kutta method of order four. [8]
5. Let $\boldsymbol{\dfrac{7x^2-2x+1}{(x-1)(x-2)(x-4)}=\dfrac{A}{x-1}+\dfrac{B}{x-2}+\dfrac{C}{x-4}}$.
Use Lagrange's interpolation to find the values of A, B, C. [4]