Birla Institute of Technology & Science, Pilani

Mid Semester Exam (Closed Book) Ist Semester 2023 – 2024
Course Name : Numerical Analysis (MATH F313) Date:

Max. Time: 90 Minutes **Max. Marks: 70**

Note: Use four significant digits with rounding wherever not mentioned. **Start answering** each question on a **fresh page**.

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1. Find the value of the polynomial
$P(x) = 5.9999x^2 + 3.0004x^4 - 2x^3 - 6.9999x + 11.00008$ at $x = 3.1402$, using the process which involves lesser number of operations (justify your process). [6]

2. Using the method of quadratic convergence, find the point of intersection of $y = 3x$ and $y = \cos x + 1$ correct up to four digits after decimal in normalize form. Take the initial approximation as $x_0 = 0.5$.(use five significant digits with rounding) . [6]

3. To Solve the non-linear equation f(x)=0, following iteration is used.

$$x_0 = 5, \quad x_{n+1} = \frac{1}{16}(x_n^4 - 8x_n^3 + 128x_n - 192) \quad n = 0, 1, 2, 3, \ldots\ldots .$$

Find the order of convergence of the above iteration at r=4, where f(r) =0. Justify your answer. [6]

4. Find the upper bound of the error in interpolating the values of $f(x) = 2e^{(3(x-1)/4)}$ at $x = 2$ based on the values at $x =$ 0, 1, 3, 4, 6. **[6]**

5. Perform Gauss-Elimination (G-E) with scaled partial pivoting, storing multipliers and pivoting vector on the following 3x3 matrix.

$$A = \begin{bmatrix} 3 & 2 & 100 \\ -1 & 3 & 100 \\ 1 & 2 & -1 \end{bmatrix}$$

Then, solve $AX=b$ where $b=(105, 102, 2)^T$ using forward and backward substitution. Hence, find the value of determinant of A. **[14]**

6. Using Newton's method, reduce the nonlinear system:

$$2x^3 - 4y^2 + z^2 + 3y = 1$$
$$5x^2 + 2y^3 - 2z^2 - 2x = 5$$
$$2x^2 yz = 3$$

to a system of linear equations in h_1, h_2 and h_3 to obtain the solution: $x = 1 + h_1$, $y = -1 + h_2$ and $z = -1 + h_3$

Hence, perform one iteration of Gauss-Seidel method to find the solution of resulting system in h_1, h_2 and h_3 with initial vector $(0,1,1)^T$ so that the iteration scheme converges to true solution. **[12]**

7. Derive Newton's divided difference interpolating formula to approximate a function using x_0, x_1,…….x_n, distinct (n+1) point. **[6]**

8. State and prove Fixed point theorem. **[14]**