Birla Institute of Technology & Science, Pilani

Comprehensive Exam **I Semester 2023-24**

Course Name: Numerical Analysis (MATH F313) **Date:**

Max. Time: 90 Min. **Max. Marks: 46**

Note: 1. The comprehensive question paper consists of two parts, Closed Book and Open Book. Attempt questions of Closed Book and Open Book in two separate answer sheets provided.

2. Each subpart of a particular question should be in continuation.
3. Write on top right corner of two answer books Closed Book and Open Book.
4. Use four significant digits with rounding in all calculations where not specified.

(Closed Book)

1. Derive three point Gauss-Chebyshev formula **with error term** and then using the derived formula without error term, evaluate $\int_0^2 \frac{xe^x}{\sqrt{x(2-x)}}dx$. [10]

2. (i) Using Divided differences (fitting cubic polynomial), derive the 4th order Adams-Bashforth-Moultan **predictor formula with error term** to find $y(x_{n+1})$as a solution of $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ (with spacing h). [10]

   (ii) Using the above predictor formula, find y(0.8) for the IVP:
   $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1, y(0.2) = 1.2, y(0.6) = 1.976, h = 0.2$, (Use Euler's method for finding the missing values.) [3]

3. Find the approximate values of *x* for which the function *f* has maxima & minima, where *f* satisfy the following data:

x	-1	0	1	2
f(x)	4	0	-6	-8

   (Use polynomial approximation by Newton's forward). [6]

4. Prove that the Jacobi's method to solve *AX=b* converges, if the coefficient matrix is strictly diagonally dominant. [5]

5. For the following initial value problem

$$\boldsymbol{y''' + 3y^2y'' - 2yy' + 5y = 2x,}$$
$$\boldsymbol{y(0) = 0,\ y'(0) = -1, y''(0) = 1,}$$

find the approximate value of $y(0.2),\ y'(0.2),\ y''(0.2)$ taking h=0.2 using Runge-Kutta method of order four. [8]

6. Using Power method, find the dominant Eigen value and the corresponding eigenvector for the following matrix [take $x^0 = (1,1,1)^T$]. (Perform four iterations only) [4]

$$\begin{bmatrix} 2 & 4 & -6 \\ 4 & 2 & -6 \\ -6 & -6 & -15 \end{bmatrix}$$

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Comprehensive Exam (Open Book) **I Semester 2023-24**

Course Name: Numerical Analysis (MATH F313) **Date:**

Max. Time: 90 Min. **Max. Marks: 44**

Note: 1. Use four significant digits with rounding in all calculations where not specified.

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1. By applying the second order finite difference method to the following boundary value problem

$$y''' - x^2 y' = 2xy,$$

with $y'(0) = 1, y(1) + y'(1) = -1,\ y''(0) = -2,$

obtain a system of linear equations in terms of y(0), y(0.5), and y(1) **with integer elements of coefficient matrix with no common factor**. Hence, solve the resulting system using forward and backward substitution with scaled partial pivoting, storing multiplier and pivoting vector. [15]

2. Using three point Gauss-Legendre formula, evaluate $\int_0^{\infty} \frac{1}{(1+x^2)^2(1+(\tan^{-1}x)^2)}dx$. [7]

3. Applying the Galerkin,s method to find the solution of the equation $-\frac{d^2y}{dx^2} + y = 5$ with boundary conditions $y'(0) = 2$ and $y'(1) = 4$ taking trial function as a polynomial of degree four: $c_0 + c_1 x + c_2 x^2 + c_3 x^2 (x-1)^2$ with one arbitrary constant and passing through the origin.[11]

4. (a) Compute the following divided difference table by writing the missing entries:

x_i	$f[x_i]$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_i,..,\ldots,x_{i+3}]$	$f[x_i,..,\ldots,..,x_{i+4}]$
1	…..				
		3			
1	…..		………..		
		……		**0**	
1	…..		………..		………….
		…….		…..	
3	**18**		**3**		
		13			
3	…..				

Hence, using this table, find $f'(3), f''(1)$

(b) Further, using this table, find interpolating polynomial of appropriate degree for this data and approximate *f(2).* [7+4]