BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Second Semester 2022 – 23
MATH F341: Introduction to Functional Analysis
Mid Semester Examination

Duration: 90 mins **Date: 17/03/2023** **MM : 70**

Q1. Let X be a non-zero normed linear space. Then show that X is a Banach space if and only if the set $S = \{x \in X: \|x\| = 1\}$ is complete. [15]

Q2. Let T be a linear transformation from a Banach space X into a Banach space Y such that graph of T is closed. Then show that T is continuous. [15]

Q3. Prove that for every x in a normed linear space X,

$$\|x\| = \sup_{0 \neq f \in X^*} \frac{|f(x)|}{\|f\|}.$$

If $x_0 \in X$ be such that $|f(x_0)| \leq c$ for all $f \in X^*$ with $\|f\| = 1$, then using the above result show that $\|x_0\| \leq c$. [10+5=15]

Q4. Let (x_n) be any sequence in an inner product space X such that $\|x_n\| \to \|x\|$ and $\langle x_n, x \rangle \to \langle x, x \rangle$, $x \in X$. Then prove that $x_n \to x$. [5]

Q5. Let S be any subset of a Hilbert space H. Then prove the following:

(i) $S^\perp = S^{\perp\perp\perp}$.

(ii) $S \cap S^\perp = \{0\}$, if S is a subspace of H.

(iii) $H^\perp = \{0\}$ and $H = \{0\}^\perp$. [5x4=20]

END