Birla Institute of Technology and Science, Pilani

2nd Semester 2016-17

Differential Geometry (MATH F342)

Mid-semester Test (Closed book)

Max. Time: 90 mins **Max. marks : 70**

Instructions : 1) Use arrow notation on top of all vectors. 2) All notations used in question paper are standard. 3) Explain all the steps.

1. A disk of radius 1, initially having center at (2, 0), rolls in anticlockwise direction over a disk of radius 1 having center (0, 0). Find the parametric equation of the curve traced by point P of the rolling disk, initially having coordinates $\left(\frac{3}{2},0\right)$. Also, find all the points where this parametrized curve is not regular. [10]
2. Obtain a unit speed reparameterization of the curve $\vec{r}(t)=\left(e^t\cos t, e^t\sin t, e^t\right),\ 0<t<1.$ [10]
3. Define the torsion of a smooth unit speed curve. Prove that the torsion of a unit speed curve $\vec{\gamma}(s)$ is given by $\tau(s)=\dfrac{(\vec{\gamma}'\times\vec{\gamma}'')\cdot\vec{\gamma}'''}{\|(\vec{\gamma}'\times\vec{\gamma}'')\|^2}$. [10]
4. For a smooth unit speed curve $\vec{\gamma}(s)$ whose curvature $\kappa=\kappa(s)$ and torsion $\tau=\tau(s)$ are both non-vanishing, show that $\left(\dfrac{d\vec{b}}{ds}\times\dfrac{d^2\vec{b}}{ds^2}\right)\cdot\dfrac{d^3\vec{b}}{ds^3}=\tau^5\dfrac{d\left(\kappa/\tau\right)}{ds}$, where the vector $\vec{b}(s)$ is a unit binormal vector to $\vec{\gamma}(s)$. [10]
5. Define involute of a smooth plane unit speed curve. Find the involute $\vec{i}(t)$ of $\vec{\gamma}(t)=(t-\sin t, 1-\cos t),\ 0<t<\pi$. [10]
6. Let $\vec{\gamma}(t)=(t,\sin t), t\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$ and let for a real number λ, $\vec{\gamma}^{\lambda}(t)=\vec{\gamma}(t)+\lambda\vec{n}_s(t), t\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$ be the curve parallel to $\vec{\gamma}(t)$. Find all the positive values of λ such that $\vec{\gamma}^{\lambda}(t)$ is regular. [10]
7. Define a general helix. Let $\vec{\gamma}(s)$ be a smooth unit speed curve with positive curvature $\kappa(s)$ and positive torsion $\tau(s)$ and $\vec{\gamma}^*(s)=\vec{b}(s)$, the unit binormal of $\vec{\gamma}(s)$. Find $\dfrac{d\kappa^*}{ds}$ for the curvature $\kappa^*(s)$ of $\vec{\gamma}^*(s)$.

   [10]

---------------------------------All the Best---