Birla Institute of Technology and Science, Pilani

2nd Semester 2017-18

Differential Geometry (MATH F342)

Mid-semester Test (Closed book)

Max. Time: 90 mins **Max. marks : 70**

Instructions : 1) Use arrow notation on top of all vectors. 2) All notations used in question paper are standard. 3) Explain all the steps.

1. A disk of radius 1, initially having center at (0,1), rolls in anticlockwise direction over X-axis. (A) Find the parametric equation of the curve traced by point P of the rolling disk, initially having coordinates $\left(0,\frac{3}{2}\right)$. (B) Find all the points where this parametrized curve is not regular. (C) Find a level curve containing the image of this parameterized curve for one roll of the disk. [4+3+3]
2. The equation of a curve C in polar coordinates is $r = \sin\theta\tan\theta,\ 0 < \theta < \pi/2$. Find the parameterization of C in terms of parameter θ. Show that it has a reparameterisation $\vec{\gamma}^*(t) = \left(\frac{1-t}{2}, \frac{(1-t)^2}{\sqrt{4-4t^2}}\right), 0 < t < 1.$ [10]
3. Find the formula for the curvature of a curve with equation $r = f(\theta)$ in polar cordinates. Hence find the curvature of the cardioid with equation $r = 1 + \cos\theta$. [10]
4. Define the signed normal and the signed curvature of a smooth unit speed plane curve. Let $\vec{\gamma}(s)$ be a smooth unit speed plane curve and let $\vec{t}(s), \overrightarrow{n_s}(s)$ denote respectively the unit tangent and signed normal of $\vec{\gamma}(s)$. Show that $\frac{d\overrightarrow{n_s}}{ds}(s) = -\kappa_s(s)\vec{t}(s)$. [8]
5. Show that the evolute of the parabola $\vec{\gamma}(t) = (t, t^2)$ satisfies the equation $27X^2 = 16\left(Y - \frac{1}{2}\right)^3$. [12]
6. Let $\vec{\gamma}(t) = (6t, 3t^2, t^3), t \in \mathbb{R}$. Find its curvature $\kappa(t)$ and torsion $\tau(t)$. Hence verify if $\vec{\gamma}(t)$ is a general helix or not. [10]
7. For a smooth unit speed curve $\vec{\gamma}(s)$ with non-vanishing curvature in $\mathbb{R}^3$, let $\vec{\gamma}^*(s) = \vec{n}(s)$, the unit principal normal to $\vec{\gamma}(s)$. Is $\vec{\gamma}^*(s)$ regular? If yes, then find the curvature $\kappa^*(s)$ of $\vec{\gamma}^*(s)$. [10]