BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

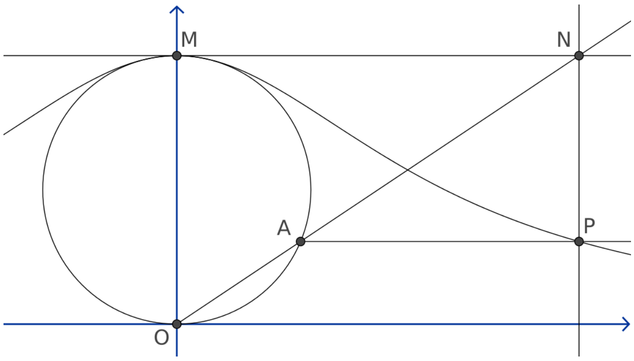
Second Semester 2022-23

MATH F342 : Differential Geometry

Mid Semester Examination (Closed Book)

March 13, 2023 **Max. Marks: 60** **Max. Time: 90 Minutes**

1. Consider a circle of radius one.
   Let A be a point on the circle and N be the point of intersection of tangent through M and the line passing through O and A. If the point P is the point of intersection of the line passing through A and parallel to the line MN, and the line passing through N and perpendicular to the line MN.



   Find the parametrization for the locus of the point P. **[8]**
2. Show that the radius of curvature ρ and radius of torsion σ of the curve $\vec{r}(u) = (\cos u, \sin u, \cos 2u)$ at $u = \pi/4$ satisfy $\rho = 6\sigma$. **[8]**

3. Prove that the tangents to two different evolutes corresponding to two constants c_1 and c_2 drawn from the same point of the given curve are inclined to each other at a constant angle $c_1 - c_2$(Recall that the parametrization for the evolute is $\vec{R} = \vec{r} + \rho\vec{n} + \rho\tan(\psi + c)\vec{b}$). **[6]**

4. Show that the necessary and sufficient condition that a curve be a helix is that
$$[\vec{r}'', \vec{r}''', \vec{r}^{(iv)}] = 0.$$ **[8]**

5. Discuss the nature of the points on the surface $\vec{r}(u, v) = (u, v, \sqrt{1 - u^2 - v^2})$. **[6]**

6. Determine the angle between the parametric curves of the surface
$$\vec{r}(u, v) = (u \sin\alpha \cos v, u \sin\alpha \sin v, u \cos\alpha).$$ **[8]**

7. Show that the tangent plane of one-sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ at point $(x, y, 0)$ is parallel to the z-axis. **[8]**

8. Determine $\phi(v)$ so that the surface given by
$$x = u\cos v,\ y = u\sin v,\ z = \phi(v)$$
shall be isometric to the surface of revolution. **[8]**

*** All The Best ***