BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
Second Semester 2022-23
MATH F342 : Differential Geometry
Comprehensive Examination (Closed Book)

May 08, 2023 | **Max. Marks: 80** | **Max. Time: 180 Mintues**

Name: | **ID No.:**

Max marks.: 30 | **PART A** | Max. Time: 60 Minutes

1. Fill in the blanks **[6× 3=18]**

(a) The equation of the tangent plane at $(1, 1, 0)$ on the surface $\vec{r}(u, v) = (u, v, u^2 - v^2)$ is ——.

Ans:

(b) The principal radii of curvature of the surface with its first fundamental form and second fundamental form as $I = du^2 + (u^2 + a^2)dv^2$ and $II = -\frac{a}{\sqrt{a^2+u^2}}dudv$, respectively, where $a \in \mathbb{R}$, are given by ——.

Ans:

(c) The parameter of the distribution of the helicoid $\vec{R}(u, v) = (v \cos u, v \sin u, \alpha u)$ is ——.

Ans:

(d) The direction coefficients of the curve which makes an angle $\pi/2$ with the curve $v =$ constant on the surface with first fundamental form as $a^2du^2 + a^2 \sin^2 u dv^2$ is given by ——.

Ans:

(e) The absolute value of the geodesic curvature of the curve $v = \pi$ on the surface $\vec{r}(u, v) = (\cos u \cos v, \sin u \cos v, \sin v)$ is ——.

Ans:

(f) The orthogonal trajectory of the family of curves $du + 2(u^2 + 1)dv = 0$ on the surface $\vec{r}(u, v) = (u \cos v, u \sin v, 1 - v)$ is given by——.

Ans:

2. State whether the following statements are TRUE or FALSE. **[6×2=12]**

(i) Let $\vec{\gamma}(t)$ be a unit speed curve in $\mathbb{R}^3$, and assume that its curvature $\kappa(t)$ is non-zero for all t. Define a new curve $\vec{\delta}$ by $\vec{\delta}(t) = \frac{d\vec{\gamma}(t)}{dt}$. If s is the arc-length parameter of $\vec{\delta}$, then $\frac{ds}{dt} = \frac{d\kappa}{dt}$.

(ii) Let $\vec{\gamma}$ be a regular curve. The involute of the evolute of $\vec{\gamma}$ is a parallel curve of $\vec{\gamma}$.

(iii) The surface with the first fundamental form as $a^2 \cosh^2 v du^2 + a^2 \cosh^2 v dv^2$ and the second fundamental form as $a^3 \cosh^2 v du dv$ is a minimal surface.

(iv) The value of the Gaussian curvature is independent of the parameters chosen.

(v) When the lines of curvature are parametric curves then $\vec{N}_1 = \frac{L}{E}\vec{r}_1$.

(vi) The torsion at a point of an asymptotic line on a surface is $\pm\sqrt{K}$.

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Max marks.: 50 | **PART B** | **Max. Time: 120 Minutes**

Instructions : 1) Use arrow notation on top of all vectors. 2) All notations used in question paper are standard. 3) Explain all the steps.

1. For a curve C in a plane, the pedal of C with respect to a point P in the plane is the locus traced by the foot A of the perpendicular from P to the tangent to C at a point Q varying on C. Find a parametrization of the pedal of the unit circle $\vec{r}(t) = (\sin t, \cos t);\ 0 \leq t < 2\pi$ from the point $P = (-2, 0)$ in the plane. **[6]**

2. Show that evolute of a parametrized curve $\vec{r}(t) = (x(t), y(t))$ has parametric equation

$$\left(x + \frac{y'(x'^2 + y'^2)}{y'x'' - x'y''}, y - \frac{x'(x'^2 + y'^2)}{y'x'' - x'y''}\right).$$

   Hence show that evolute of cardioid with polar equation $r = 1 + \cos\theta,\ 0 < \theta < \pi$ is again a cardioid. **[6]**

3. Show that if at a point P of the surface, ϕ denotes the angle between the principal normal $\vec{n}$ of the curve $\vec{r}$ and the surface normal $\vec{N}$, then $\kappa_n = \kappa \cos\phi$. **[5]**

4. Let $\kappa_1,\ \kappa_2$ denote the principal curvatures of the surface $\vec{\sigma}(u,v) = (u\cos v, u\sin v,\ u^2 + v)$, $u > 0,\ 0 \leq v \leq 2\pi$. Find $\kappa_1\kappa_2$ at all the points of the curve $\vec{r}(v) = \vec{\sigma}(1, v), 0 \leq v \leq 2\pi$. **[6]**

5. Determine the asymptotic lines for the Enneper's surface

$$\vec{r}(u,v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + u^2v, u^2 - v^2\right).$$ **[6]**

6. Find the curve conjugate to the parallels $u =$ constant on the sphere. **[5]**

7. Prove that two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$ are orthogonal on a surface iff $ER - 2QF + GP = 0$. **[5]**

8. For the ruled surface formed by the principal normals of a curve, find the parameter of distribution and hence obtain its Gaussian curvature. **[6]**

9. Prove/disprove the following

$$H[\vec{N},\ \vec{N}_1, \vec{r_1}] = EM - FL \text{ and } H[\vec{N},\ \vec{N}_1, \vec{r_2}] = FM - GL.$$ **[5]**

*** All The Best ***