Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2017-2018, MATH F343 (Partial Differential Equations)
End Semester Examination

Time: 180 Min. **Date: May 14, 2018 (Monday)** **Max. Marks: 80**

1. Write solution of each question on fresh page.
2. All questions are compulsory and carry equal marks.

CLOSED BOOK QUESTIONS

Q. 1 Write solution of wave equation in one dimension by using suitable conditions. Hence, find the displacement $u(x,t)$ in a tightly stretched string with fixed end points $x = 0$ and $x = \pi$, which is released from the initial position $u(x) = 4\sin^3 x$.

Q. 2 Find solution of the Laplace equation (two dimensional) in polar coordinates.

Q. 3 An elastic rectangular membrane is stretched with fixed edges along $x = 0$, $x = a$, $y = 0$ and $y = b$. Find displacement of the membrane from xy-plane subject to the initial position $f(x,y)$ and initial velocity $g(x,y)$.

Q. 4 Find the heat flow in a rectangular volume where the faces $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$ are kept at zero temperature, and the initial temperature is $f(x,y,z)$.

OPEN BOOK QUESTIONS

Q. 1 Transform the partial differential equation

$$y + 2zq = q(4xp + yq)$$

to Clairaut's form, and hence find its complete solution.

Q. 2 Solve $(x^2 z_{xx} - 4xyz_{xy} + 4y^2 z_{yy} + 6yz_y)z = x^3 y^4$.

Q. 3 Use suitable Fourier transform to find heat flow in a semi-infinite rod where the finite end $x = 0$ is kept at zero temperature, and the initial temperature vanishes except that it is non-zero and constant till a finite length from $x = 0$.

Q. 4 Use Laplace transform to solve the following IVBP:

$$u_{tt} = c^2 u_{xx} + \sin(\pi x),\ 0 < x < 1,\ t > 0,$$

$$u(x,0) = 0,\ u_t(x,0) = 0,\ u(0,t) = 0,\ u(1,t) = 0.$$

END