Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2017-2018, MATH F343 (Partial Differential Equations)
Mid Semester Examination (Closed Book)

Time: 90 Min.　　　　　**Date: March 10, 2018 (Saturday)**　　　　　**Max. Marks: 60**

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1. Write solution of each question on fresh page.

2. All questions are compulsory and carry equal marks.

3. Write **END** in the answer sheet just after the final attempted solution.

Q. 1 Construct a partial differential equation (PDE) of the form $Pz_x + Qz_y = R$ by eliminating the arbitray function ϕ from $\phi(u, v) = 0$, where P, Q, R, u and v are functions of x, y and z. Hence find the PDE corresponding to $\phi(x + yz, x^2 + y^2 - z^2) = 0$.

Q. 2 Find a singular solution of the PDE

$$2xz - px^2 - 2qxy + pq = 0.$$

Q. 3 Transform the following PDE into canonical form:

$$yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0.$$

Q. 4 Find the general solution of the PDE

$$u_{xx} - u_{xy} - 2u_{yy} + 2u_x + 2u_y = e^{2x+3y} + xy + \sin(2x + y).$$

Q. 5 Determine the solution of the PDE

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = \frac{\partial^2 u}{\partial t^2}$$

subject to the following conditions:

$$u(r, 0) = r, \ 0 \leq r < \infty,$$

$$u_t(r, 0) = e^{r^2}, \ 0 \leq r < \infty,$$

$$u(0, t) = 2, \ 0 \leq t < \infty.$$

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