Birla Institute of Technology & Science, Pilani

Second Semester 2021-2022

MATH F343 (Partial Differential Equations)

Comprehensive Examination Part A (Closed Book)

Time: 90 Min. **Date: May 14, 2022 (Saturday)** **Max. Marks: 40**

1. Write PART-A on the top of your answer sheet.
2. While answering, justify your steps. Just writing the final answer will receive no credit.
3. Write **END** after the last attempted solution.
4. You can submit PART-A any time between 9 : 00 AM and 10 : 00 AM to start PART-B. Ideally you should not spend more than 90 minutes on part-A.

1. Show that the problem

$$\begin{aligned} \Delta u &= 0 \quad \text{in } \Omega \\ u &= f(x) \ \text{ on } \partial\Omega, \end{aligned}$$

has a unique solution(if it exists), where Ω is a bounded domain in $\mathbb{R}^3$. **[5]**

2. Find the general solution of the problem

$$u_{xx} - 4u_{xy} + 4u_{yy} = e^{2x+y}. \quad \textbf{[6]}$$

3. Verify the compatibility condition for the existence of a solution and determine the solution to the initial boundary value problem **[5]**

$$\begin{aligned} u_{tt} = 16u_{xx} \quad & 0 < x < \infty,\ t > 0 \\ u(x,0) = \cos x \quad & 0 \le x < \infty, \\ u_t(x,0) = x^2 \quad & 0 \le x < \infty, \\ u(0,t) = 0 \quad & t \ge 0. \end{aligned}$$

4. Find the solution to the problem $\Delta u = 0$ in the disk $r < a$ under the boundary condition $\frac{\partial u}{\partial r} - hu = \theta$ when $r = a$, where h is a constant. Assume that the solution is bounded in the disk and $u(r,\theta) = u(r,\theta + 2\pi)$. **[12]**

5. Solve **[12]**

$$\begin{aligned} u_t = u_{xx} \quad & 0 < x < 10,\ t > 0, \\ u(0,t) = 20,\ u(10,t) = 40 \quad & t \ge 0, \\ u(x,0) = 2x + 20 \quad & 0 \le x \le 10. \end{aligned}$$

End of Paper