# Birla Institute of Technology & Science, Pilani
## MATH F343 (Partial Differential Equations)
## Mid-Semester Examination (Closed Book)

**Max. Marks: 70** **March 12, 2022** **Time: 90 Min.**

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1. Let $a, b, c, d \in \mathbb{R}$ be such that $c^2 + d^2 \neq 0$. Find a general condition on constants $a, b, c, d$ so that the Cauchy problem

$$au_x + bu_y = e^{x+y}, \ x, y, \in \mathbb{R}$$

$$u(x, 0) = 0 \text{ on } cx + dy = 0,$$

admits a unique solution. **[4]**

2. Find complete integral of the PDE

$$pqz = p^2(xq + p^2) + q^2(yp + q^2).$$ **[4]**

3. Use the method of separation of variables to find a solution to

$$x^2v_x^2 + y^2v_y^2 = 1.$$ **[5]**

4. Solve

$$u_{tt} - c^2u_{xx} = xe^t, \ u(x, 0) = \sin x, \ u_t(x, 0) = 0.$$ **[6]**

5. Reduce

$$u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0$$

into the canonical form. **[8]**

6. Use the method of characteristics to solve the Cauchy problem (Do not use Charpit's method)

$$u_x^2 + u_y^2 = 1, \ u = 0 \text{ along } x^2 + y^2 = 1.$$ **[14]**

7. (a) Find general solution of $u_{tt} - c^2u_{xx} = 0, \ -\infty < x < \infty, \ t > 0.$ **[5]**

   (b) Consider the problem

$$\begin{array}{ll} u_{tt} - u_{xx} = 0 & 0 < x < 2, \ t > 0, \\ u(x, 0) = x^2(2 - x)^2, \ u_t(x, 0) = 0 & 0 \leq x \leq 2, \\ u_x(0, t) = 0, \ u_x(2, t) = 0 & 0 \leq t < \infty. \end{array}$$

   Without using the method of separation of variables, find $u\left(\frac{3}{2}, 4\right)$. **[12]**

8. Find a solution to the problem:

$$\begin{array}{ll} u_{tt} - u_{xx} = 0 & 0 < x < \infty, \ t > 0, \\ u(x, 0) = x^2, \ u_t(x, 0) = 0 & 0 \leq x < \infty, \\ u_x(0, t) + 2u(0, t) = 0 & 0 \leq t < \infty. \end{array}$$ **[12]**

**End of Paper**