Birla Institute of Technology & Science, Pilani

Mid Semester Examination (Closed Book), Second Semester 2022 - 2023

Partial Differential Equations (MATH F343)

Date: 15^{th} **March, 2023** **Max. Time: 90 Minutes** **Max. Marks: 70**

Note. Answer all questions with proper justification. Start answering each question on a fresh page.

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1. Obtain a first-order PDE
   (a) of all spheres of radius r having center in the xy-plane. **[5]**
   (b) by eliminating f from $z = x^n f(y/x)$. **[5]**

2. Use Charpit's method to find a complete integral of $(p^2 + q^2)x = pz$. **[10]**

3. Using Jacobi's method find the complete integral of $z + 2u_z - (u_x + u_y)^2 = 0$. **[10]**

4. Decide whether the equation $3u_{xx} + 4u_{xy} - \frac{3}{4}u_{yy} = 0$ is elliptic, parabolic, or hyperbolic. Then reduce the equation into canonical form. Hence, show that the general solution is given by $u = f(y - \frac{3}{2}x) + g(y + \frac{1}{6}x)$. **[10]**

5. Solve $(D^2 - DD' - 2D)z = \sin(3x + 4y)$. **[10]**

6. Consider the following IVP

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, \quad -\infty < x < \infty,\ t > 0, \\ u(x,0) &= f(x), \quad -\infty < x < \infty, \\ u_t(x,0) &= g(x), \quad -\infty < x < \infty. \end{aligned}$$

   (a) If $f(x)$ and $g(x)$ are periodic functions of period $2L$, then show that $u(x,t)$ is periodic in x of period $2L$. **[5]**
   (b) If $f(x)$ and $g(x)$ are odd functions, then show that $u(x,t)$ is odd in x. **[5]**

7. Determine the solution of the following IBVP

$$\begin{aligned} u_{tt} &= 16u_{xx}, \quad 0 < x < \infty,\ t > 0, \\ u(x,0) &= \sin x, \quad 0 \le x < \infty, \\ u_t(x,0) &= x^2, \quad 0 \le x < \infty, \\ u(0,t) &= 0, \quad t \ge 0. \end{aligned}$$

[10]