Birla Institute of Technology & Science, Pilani

Comprehensive Examination (Closed Book), Second Semester 2022 - 2023

Partial Differential Equations (MATH F343)

Date: 13^{th} **May, 2023** **Max. Time: 70 Minutes** **Max. Marks: 35**

Note. Answer all questions with proper justification. Start answering each question on a fresh page.

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1. Find the complete integral of $p^2q(x^2+y^2) = p^2+q$. **[9]**

2. By using an appropriate substitution, convert the following PDE

$$x^2 z_{xx} - 4xyz_{xy} + 4y^2 z_{yy} + 6yz_y = x^3y^4,$$

into a PDE with constant coefficients. Hence, find its general solution. **[9]**

3. Using Fourier transforms show that $\int_0^\infty e^{-a^2x^2}\cos bx\,dx = \frac{\sqrt{\pi}}{2a}e^{-b^2/4a^2}$. **[9]**

4. Using Laplace transforms solve the following problem:

$$\begin{aligned} u_{tt} &= u_{xx} + 1,\ 0 < x < \infty,\ t > 0, \\ u(x,0) &= 0,\ u_t(x,0) = 0, \\ u(0,t) &= 0,\ u_x(x,t) \to 0,\ \text{as } x \to \infty. \end{aligned}$$

[8]