Birla Institute of Technology & Science, Pilani

MATH F353 (Statistical Inference and Applications)
Second Semester 2021-2022
Mid Semester Examination (Open Book)

Time: 90 Minutes **Date: March 10, 2022 (Thursday)** **Max. Marks: 60**

Note: Only class-notes are allowed. Notations and symbols have their usual meaning. Define random variables as and when required. Assumptions made if any, should be stated clearly at the beginning. Start new question on fresh page. Moreover, answer each subpart of a question in continuation.

1. Let X be a discrete random variable with $P(X=0)=\frac{2}{3}\theta$, $P(X=1)=\frac{1}{3}\theta$, $P(X=2)=\frac{2}{3}(1-\theta)$, and $P(X=3)=\frac{1}{3}(1-\theta)$, where $0 \leq \theta \leq 1$ is a parameter.
   (a) Based on a random sample of size n from the population, find the method of moment estimator for θ. Is it unbiased? Is it consistent?
   (b) Again, based on a random sample of size n from the population, find the MLE estimator for θ, and compute its approximate variance using asymptotic theory. **[15]**

2. Suppose that $X \sim Bin(n,\theta)$, n known and $0<\theta<1$.
   (a) What is the lower bound for the variance of an unbiased estimator for $g(\theta)=\theta(1-\theta)$?
   (b) With proper justification, provide an unbiased estimator T for $g(\theta)$ in the form of $T=hX+kX^2$; h and k are constants.
   (c) Is T obtained above asymptotically efficient? **[15]**

3. Let $X_1, X_2, ..., X_n$ be a random sample from a population with pdf
$$f(x,\theta)=\theta x^{(\theta-1)}, 0<x<1, \theta>0.$$
   (a) Does $f(x,\theta)$ belong to the exponential family?
   (b) Is $\sum_1^n X_i$ sufficient for θ?
   (c) Find a complete sufficient statistic for θ.
   (d) Find the MLE of θ. Is it a function of the sufficient statistic? **[15]**

4. Answer the following problems with proper justification.
   (a) Consider the family of uniform distributions on $(-\theta,\theta), \theta>0$. Is the family complete?
   (b) Let $X_1, X_2, ..., X_n$ be a random sample from a $N(\mu,\mu)$ family; $\mu>0$. Find a minimal sufficient statistic for μ.
   (c) Let $X_1, X_2, ..., X_n$ be a random sample from a location family. Prove or disprove that $(M-\bar{X})$ is an ancillary statistic, where M is the sample median. **[15]**

END