

# Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2022-2023, MATH F421 - CS F451

Mid-semester Exam (Closed Book)

Time: 90 Minutes

Date: November 2, 2022

Max. Marks: 70

Calculators are not allowed.

**Q.1** When all single digits are written on a sheet of a paper, after  $180^\circ$  rotation, the digits 0, 1 and 8 do not change meaning, the digit 6 becomes 9 and 9 becomes 6. The remaining digits lose their meaning. Find the number of 7-digit positive integers that remain the same after  $180^\circ$  rotation? [8]

**Q.2** Using generating function, solve the following Recurrence Relation [8]

$$a_{n+2} - 2a_{n+1} + a_n = 2^n; \quad n \geq 0, a_0 = 1, a_1 = 2$$

**Q.3** Using Recurrence Relation, find the number of ways to tile a  $2 \times n$  chessboard using horizontal ( $1 \times 2$ ) dominoes, vertical ( $2 \times 1$ ) dominoes, and square  $2 \times 2$  tiles, if dominoes come in 4 colors and square tiles come in 5 colors? [15]

**Q.4** A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on a vertical pole in order to communicate a signal to other ships. Using generating function, find the number of signals having at least 3 white flags or no white flags at all? [13]

**Q.5** At Ram's flower shop, Ram wants to arrange 15 different plants on 5 selves for a window display. Using Inclusion-Exclusion Principle, find, in how many ways, can he arrange them so that each shelf has at least one but no more than 4 plants? [13]

**Q.6** For  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{u, v, w, x, y, z\}$ , using Rook's Polynomial, find the number of one-to-one functions  $f : A \rightarrow B$  where  $f(1) \neq v, w$ ;  $f(2) \neq u, w$ ;  $f(3) \neq x$ ; and  $f(4) \neq v, x, y$ ? [13]