

Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2022-2023, MATH F421 - CS F451

End-semester Exam (Open Book)

Time: 75 Minutes

Date: December 23, 2022

Max. Marks: 35

Part-A

Calculators are not allowed.

- Q.1** Find the number of possible routes from the start $(0, 0)$ to the corner $(5, 4)$ for the grid in Figure 1. [7]

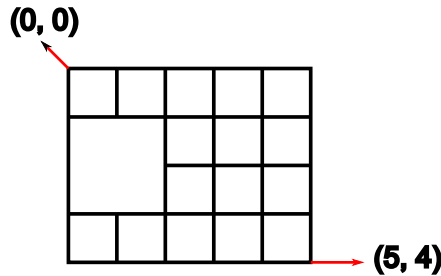


Figure 1:

- Q.2** Using exponential generating function, find the number of n -letter strings which can be constructed from the letters of the word BARBER. [8]
- Q.3** Determine a recurrence relation and initial conditions for the number of ordered partitions of n into non-zero parts with numbers 1, 2, or 5. [6]
- Q.4** Draw a board B with rook polynomial $1 + 20x + 90x^2$. [5]
- Q.5** Using Polya's Formula, find, in how many ways can we 3-color the eight regions of pinwheel in Figure 2 having 4 black, 2 gold, and 2 blue regions, provided back of each region remains grey. [9]

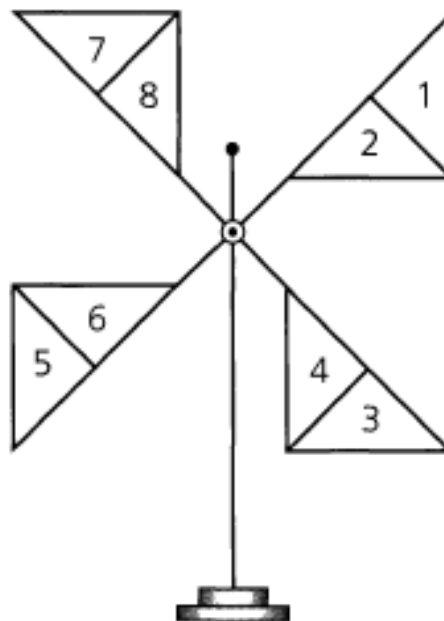


Figure 2:

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End-semester Exam (Open Book)

Time: 105 Minutes

Date: December 23, 2022

Max. Marks: 55

Part-B

Calculators are not allowed.

- Q.1** Using Inclusion-Exclusion Principle, find, how many ways are there to distribute 18 toys to six children if each child receives a toy and the 18 toys can be divided into three groups of 6, 7, 5 each, and the toys within each group are identical? [11]
- Q.2** Using Burnside lemma, find, how many ways are there to put 10 identical balls into 3 identical boxes, such that none of the boxes has more than 5 balls? (Hint: mark the boxes as 1, 2, 3 and consider symmetry group S_3). [12]
- Q.3** Let X be a set of 12 distinct numbers from 1, 2, . . . , 100. Show that there are two subsets of X each having exactly 5 distinct elements and such that the sum of their elements is the same. [8]
- Q.4** With justification, find $r(C_3, C_4)$ where C_3 is a cycle of length 3, i.e., a triangle and C_4 is a cycle of length 4. [13]
- Q.5** Compute the value for $S(n, n - 2)$. Hence, find a, b, c such that $S(n, n - 2) = C(n, a) + bC(n, c)$. [11]