BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Comprehensive Exam: Second Semester, 2020-21

Stochastic Processes and their Applications (MATH F424)

May 13, 2021

Max. Time : 120+15 mins **Max. Marks: 35**

Name :.. **ID No. :**.......................................

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1. Consider a machine that operates for a T amount of time and then fails. Once it fails, it gets repaired with a repair time R, which is also a random variable (independent of T). Let T and R both are exponentially distributed with means μ and λ, respectively. The machine is as good as new after the repair is complete. Find the probability that a machine is up at time t given that it was up at time 0 (note that the machine could have gone through many failures and repairs up to time t). [5]

2. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate with parameter λ. Given that only one occurrence of a Poisson process $N(t)$ has occurred by epoch t_0 then find $P\left[T_1 \leq x\right]$ where $0 \leq x \leq t_0$. [3]

3. Consider the process of customers arriving at a restaurant. Suppose the customers arrive in parties (or batches) of variable sizes. The successive party sizes are iid random variables and are binomially distributed with parameters n and p. The parties themselves arrive according to a Poisson process with a rate of λ per hour. Let $C(t)$ be the total number of customers arrivals up to time t, find $E\left[C(t)\right]$ and $Var\left[C(t)\right]$. [2+2]

4. Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Find $Cov\left[N\left(s\right), N(t)\right]$, $t \geq s > 0$. [5]

5. Let $\{B(t) : t \geq 0\}$ is the standard Brownian motion.

   (i) Find a function $a\left(t\right) > 0$ such that an $B(t) \in \left(-a\left(t\right), a\left(t\right)\right)$ at time $t > 0$ with probability $1/2$. [6]

   (ii) Find probability that an $B(t)$ is below zero at time 1 and above zero at time 2. [5]

6. Suppose the price (in USD) of a stock at time t (in days) is given by

$$V(t) = e^{2B(t)}, t \geq 0.$$

   where $\{B(t) : t \geq 0\}$ is the standard Brownian motion. Suppose an investor owns 500 shares of the stock at time 0. He plans to sell the shares as soon as its price reaches \$3. What is the probability that he has to wait more than 30 days to sell the stock? [7]