Birla Institute of Technology & Science, Pilani
Second Semester 2022 - 2023
Mid-Semester Examination

Course Title : Stochastic Processes and their Applications
Nature of Exam : Open Book
Marks : 30 Marks
Duration : 90 Mins

No. of Pages = 2
No. of Questions = 5

Note to Students:

1. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
2. It is an open book exam. The text book (J Medhi), the reference book (V G Kulkarni) and their photocopies are allowed.
3. Lecture slides and handwritten class notes (or photocopies) are allowed.
4. Solution manuals, previous year solutions are NOT allowed.
5. Use of a scientific calculator is allowed.

Q.1. Let $N(t)$ be a Poisson process such that

$2\,P(N(1) = 0) = 4P(N(1) = 1) + 5P(N(1) = 2)$.

Find the probability that the waiting time between the first and the second arrival is greater than 5/2. **[3]**

Q.2. A fair coin is tossed respectively until two consecutive heads, and after that the game stopes. Let N denotes the number of tosses required to stop the game.

(a) Show that this game can be modelled as a Markov chain with a four states: {H,H}, {H,T}, {T,H} and {T,T} by constructing its *Transition Probability Matrix*. **[2]**

(b) Draw the Transition graph for this process and identify its communication class. **[1]**

(c) Identify open and closed classes. **[1]**

(d) Identity the persistent and transient classes. **[1]**

(d) Find the stationary distribution. **[1]**

(e) How would you relate $P(N = n)$ (the probability that game stopes exactly in n tosses) to the Transition probability matrix? **[1]**

Q.3. Suppose that there are three independent and identically behaving machines in the shop. If a machine is up at the beginning of a day, it stays up at the beginning of the next day with probability p, and if it is down at the beginning of a day, it stays down at the beginning of the next day with probability q, where $0 < p, q < 1$ are fixed numbers. Let X_n be the number of working machines at the beginning of the nth day. Show that $\{X_n : n \geq 0\}$ is a DTMC, and find its transition probability matrix. [7]

Q.4. Let $T_i, 1 \leq i \leq k$ be k IID exponential random variables with parameter λ. Define

$$T = Max(T_i, 1 \leq i \leq k).$$

(a) Find $E[T]$ (explain all the steps). [5]

(b) Find the CDF for T. [4]

Q.5. Let $N(t), t \geq 0$ be a Poisson process with arrival rate parameter λ. Prove or disprove

$$Cov[N(t), N(t+s)] = \lambda s.$$

[4]

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