

Birla Institute of Technology & Science, Pilani

MATH F425 (Numerical Linear Algebra)

Second Semester 2022-2023

Mid-Sem Examination (Closed Book)

Time: 90 Minutes

Date: March 13, 2023 (Monday)

Max. Marks: 35

1. Notations and symbols have their usual meaning.
2. Start new question on fresh page. **Moreover, answer each subpart of a question in continuation.**
3. Write **END** at the end of the last attempted question.

Q.1(a) If A and B are symmetric matrices of order n , find the flop-counts for $C = A + B$ [3]

(i) without considering symmetricity and (ii) taking the advantages of the symmetry.

(b) Consider the system of equation $3x + y - z = 5$, $5x - 3y + 2z = 7$ and $2x - y + z = 3$. Find a permutation matrix P , lower and upper triangular matrices L and U , respectively, such that $PA = LU$. Determine the growth factor. [6]

Q.2 Consider $-x + 3y + z = 1$, $2x + y + 4z = 1$, and $4x + 2y + z = 1$. [5]

(i) Discuss the ill-conditioning of the given linear system.

(ii) If $x^* = [0.06, 0.32, 0.14]^T$ be an approximate solution of the given system, find the residual r and explain the idea of iterative refinement method to provide next better approximate solution.

Q.3 (a) Check whether matrix $A = \begin{pmatrix} 4 & 2 & 3 \\ 2 & 17 & 1 \\ 3 & 1 & 5 \end{pmatrix}$ is positive definite or not by using Cholesky method. [5]

(b) Find the QR factorization of A by using Householder matrix for [6]

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \quad b = (1, 2, 3)^T,$$

and hence, write the expression for x which solves the system of equation $Ax = b$.

Q.4 (a) If $\sigma_i > 0$ for $i = 1, 2, \dots, r$ are the singular values of a matrix A , then show that [3+5+2]

$$\sigma_{\max}(A) \leq \|A\|_F \leq \sqrt{r}\sigma_{\max}(A).$$

(b) Find the singular value decomposition ("reduced size") of matrix $A = \begin{pmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{pmatrix}$.

(c) Using SVD of A , prove or disprove that $\mathbf{Cond}_2(A^T A) = (\mathbf{Cond}_2(A))^2$.

END