

Birla Institute of Technology & Science, Pilani

MATH F432 (Applied Statistical Methods)

First Semester 2023–2024

Mid Semester Examination (Open Book)

Time: 90 Minutes

Date: October 12, 2023 (Thursday)

Max. Marks: 60

Note: Notations and symbols have their usual meaning. Assumptions, made if any, should be stated clearly. Start new question on fresh page. Moreover, answer each subpart of a question in continuation.

1. (a) Consider a random sample $\{X_1, X_2, \dots, X_n\}$ from $U[0, \theta]$ distribution. Let $U = \frac{1}{\theta} \max\{X_1, X_2, \dots, X_n\}$.

(i) Prove or disprove that $P\left[\left(\frac{\alpha}{2}\right)^{1/n} \leq U \leq \left(1 - \frac{\alpha}{2}\right)^{1/n}\right] = 1 - \frac{\alpha}{2}$. [5]

(ii) Also, prove or disprove that $P\left[\alpha^{1/n} \leq U \leq 1\right] = 1 - \alpha$. [5]

- (b) Consider a random sample $\{X_1, X_2, \dots, X_n\}$. Find k such that $(\bar{X}^2 + 2kS^2)$ is unbiased for μ^2 . [5]

2. During a total solar eclipse, the temperature drops quickly as the moon passes between the earth and the sun. The below sample data are obtained on the drop in temperature in degrees Celsius at two types of locations in southern Africa during the June 2001 eclipse:

Mountainous terrain	15	12	16	16	13	15	11	19	15		
River-level terrain	13	17	19	16	15	19	18	20	21	22	24

- (i) Construct a 99% two-sided confidence interval on $\mu_1 - \mu_2$. [8]
(ii) At $\alpha = 0.2$, is there a difference in the variances in temperature drop in these two terrains? [7]

3. A domestic turkey farmer tested three kinds of poultry feeds with the weights (in kg) of the grown turkeys in each sample given below.

Feed A	Feed B	Feed C
4.5	6.0	5.0
4.3	7.5	5.0
5.2	6.5	5.6
6.0	8.0	6.4

Test at $\alpha = 0.05$ whether there is a difference in the mean weights of turkeys consuming the different feeds. If you find a difference, use Fisher's LSD approach to determine where the difference lies. Assume the distributions are normal and variances are equal. [15]

4. (a) The estimated SLRE corresponding to advertising expenditure (x) and revenue (y) (in thousands of US dollars) is noted as $\hat{y} = 29.40 + 1.55x$. For $n = 7$, $SSR = 692$, and $SST = 1001$, use an F -test to decide whether revenue and advertisement costs are related at $\alpha = 0.05$. [5]
(b) Using the information of part (a), develop a 95% prediction interval for the ABC limited, a specific company that spends 9 thousand dollars ($x^* = 9$) on advertisement. It is given that $s_{\hat{y}^*} = 3.40$ for $x^* = 9$. [5]
(c) The estimated regression equation based on 10 observations is obtained as $\hat{y} = 29.10 + 0.56x_1 + 0.49x_2$. The values of SST and SSR are 6724 and 6216, respectively. Find SSE, R^2 , and adjusted R^2 . [5]

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