Birla Institute of Technology and Science, Pilani
I-Semester 2023-24
(Discrete Mathematical Structures) MATH F441
Mid-Semester Exams (Close Book)

Max. Marks 25   Date: 11th October 2023   Time: 90 Min.

Q.1. Factorize $f(x) = x^5 + x^4 + 1$ in $\mathbb{Z}_2[x]$ by using Berlekamp's algorithm.   [6]

Q.2. If f is a polynomial of degree n, then prove $R(f, x^n) = [R(f, x)]^n = (f(0))^n$.   [5]

Q.3. Prove or disprove that $\sqrt[m]{2}$ is irrational for all $m \geq 0$.   [5]

Q.4. An old woman goes to market and a horse steps on her basket and crashes the eggs. The rider offers to pay for the damages and asks her how many eggs she had brought. She does not remember the exact number, but when she had taken them out two at a time, there was one egg left. The same was happened when she picked them out three, four, five, and six at a time, but when she took them seven at a time they came out even. What is the smallest number of eggs she could have had? Justify. [4]

Q.5. (a) Compute $\varphi(7!)$.   [1]

(b) Find with brief justification the reminder when 5^{119} is divided by 59.   [2]

(c) A finite field F with 16 elements has how many *primitive* elements? Justify. [2]