Birla Institute of Technology and Science, Pilani

I Semester 2017-18

Comrehensive Examination (Closed Book)

MATH F444 (Numerical Solutions of Ordinary Differential Equations)

Date: 05-12-2017 **Max. Time: 180 Min.** **Max. Marks: 45**

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1. Discuss the stability of the ODE system $\mathbf{y}' = A\mathbf{y}$. [3]

2. Give the geometrical interpretation of the forward Euler method. [2]

3. Show that the trapezoidal method can be viewed as a half-step of forward Euler method followed by a half-step of backward Euler. [3]

4. For a given ODE $\mathbf{y}' = \mathbf{f}(\mathbf{y})$, consider the θ-method

$$\mathbf{y}_n = \mathbf{y}_{n-1} + h[\theta \mathbf{f}_n + (1-\theta)\mathbf{f}_{n-1}]$$

for some value θ, $0 \leq \theta \leq 1$. Find the range of θ-values such that the method is A-stable. [4]

5. Consider the two step method

$$y_n - y_{n-1} = \frac{h}{16}(9f_n + 6f_{n-1} + f_{n-2}).$$

Write the characteristic polynomials of the above method. Check if the method is consistent. Is the method 0-stable? [4]

6. Given

$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2, \quad y(0) = 1, \quad y(0.1) = 1.06, \quad y(0.2) = 1.12.$$

Use the predictor-corrector method to evaluate $y(0.3)$ accurate upto 3-digits after decimal. [4]

7. Describe

   a. the quasilinearization procedure to construct a sequence of linear BVP for solving nonlinear BVP, [4]

   b. the extrapolation technique to accelerate the convergence of the numerical methods. [4]

8. Consider the scalar Dirichlet problem

$$-\epsilon u'' + au' = q(t),$$
$$u(0) = b_1, \quad u(1) = b_2,$$

where $a \neq 0$ is a real constant and $0 < \epsilon << 1$. An upwind method is obtained by replacing the discritization of u' with forward or backward Euler, depending on $sign(a)$:

$$\frac{\epsilon}{h^2}(-u_{n-1} + 2u_n - u_{n+1}) + \frac{a}{h}\phi_n = q(t_n),$$
$$\phi_n = \begin{cases} u_{n+1} - u_n, & a < 0, \\ u_n - u_{n-1}, & a\mathbf{ge}0. \end{cases}$$

Show that A is diagonally dominant for all $R = \frac{|a|h}{\epsilon}\mathbf{ge}0$. [3]

9. The following equations describe a chemical reaction

$$\begin{aligned} C' &= K_1(C_0 - C) - R, \\ T' &= K_1(T_0 - T) + K_2 R - K_3(T - T_C), \\ 0 &= R - K_3 e^{-K_4/T} C, \end{aligned}$$

where the unknowns are the concentration $C(t)$, the temperature $T(t)$,and the reaction rate per unit $R(t)$. The constants K_i,, $i = 1, 2, 3, 4$ and the functions C_0 and T_0 are given. Assuming that the temperature of the cooling medium $T_C(t)$ is also given, what is the index of this DAE? Is it in Hessenberg form? [3]

10. State and prove Cea's Lemma. [2]

11. Consider the two point boundary value problem

$$-y'' + xy = x^3 - 2 \quad \text{in } (0, 3), \quad y(0) = 0, \quad y(3) = 9.$$

Obtain the approximate solution using finite element method with $h = 1$, $p = 1$ by showing the following steps

(a) variational formulation of the given BVP [2]

(b) Comment on the existene of the weak solution [1]

(c) Galerkin formulation [1]

(d) construction of matrices [3]

(e) Approximate solution in the interval $[0, 3]$. [2]

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