Birla Institute of Technology and Science, Pilani

I Semester 2017-18

Mid-Semester Examination (Closed Book)

MATH F444 (Numerical Solutions of Ordinary Differential Equations)

Date:Oct. 10, 2017 Duration: 90 Minutes Max. Marks: 35

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1. Define A-stable numerical method. Draw the region of absolute stability for forward Euler method for IVP. Is forward Euler method A-stable? Justify your answer. [2+2+1]

2. Find the truncation error for the trapezoidal method for $y' = f(y), y(0) = y_0$. [5]

3. Derive the necessary order condition for s-stage Runge-Kutta method. [5]

4. Prove that backward Euler method when applied to $y' = \lambda(y - g(t))$ where $g(t)$ is bounded but otherwise arbitrary function yields
$$|y_n - g(t_n)| \to 0 \quad \text{as} \quad h_n \mathcal{R}e(\lambda) \to -\infty.$$
[5]

5. Derive a two step BDF method using order condition. [5]

6. Discuss the multiple shooting method for linear system of first order differential equation. [5]

7. Consider the BVP
$$\begin{aligned} \mathbf{y}' &= A\mathbf{y}, \\ B_0\mathbf{y}(0) + B_b\mathbf{y}(b) &= \mathbf{b}, \end{aligned}$$
where $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B_b = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The fundamental solution of the BVP is given by
$$\mathbf{\Phi}(t) = \frac{1}{\sinh b} \begin{pmatrix} \sinh(b-t) & \sinh(t) \\ -\cosh(b-t) & \cosh(t) \end{pmatrix}.$$
Show that the problem has exponential dichotomy. [5]

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