Birla Institute of Technology & Science, Pilani

First Semester 2023-2024, MATH F444 (Numerical Solutions of ODEs)

Mid-Semester Examination (Closed Book)

Time: 90 Min. **Date: October 10, 2023 (Tuesday)** **Max. Marks: 35**

1. Answer a new question on a fresh page. Moreover, answer all parts of a question in continuation.
2. Use 3 significant digit arithmetic with rounding whenever it is not specified.
3. Write **END** in the answer sheet just after the final solution.

1. (a) Consider the IVP $y' = t + y, y(0) = 1$ over the interval $[0, 1]$. Determine the priori error bound of explicit Euler's method with initial error $e_0 = 0$. **[3]**

   (b) Find and sketch the region of absolute stability of implicit Euler's method for solving the test equation $y' = \lambda y$ for $\lambda \in \mathbb{C}$. **[3]**

   (c) Write Buther Table for the Runge-Kutta scheme $y_{n+1} = y_n + \frac{h}{8}(k_1 + 3k_2 + 3k_3 + k_4)$ where $k_1 = f(t_n, y_n), k_2 = f(t_n + \frac{h}{3}, y_n + \frac{h}{3}k_1), k_3 = f(t_n + \frac{2h}{3}, y_n - \frac{h}{3}k_1 + hk_2)$ and $k_4 = f(t_n + h, y_n + hk_1 - hk_2 + hk_3)$. **[3]**

2. (a) Derive a 3rd order method of the form

$$y_{j+1} = a_1 y_{j-1} + h(b_0 y'_{j+1} + b_1 y'_j + b_2 y'_{j-1})$$

   and hence, give the order of corresponding local truncation error. **[5]**

   (b) Find the region of absolute stability for the Heun's method $y_{n+1} = y_n + h/4(k_1 + 3k_3)$ where

$$k_1 = f(x_n, y_n), \quad k_2 = f(x_n + \frac{h}{3}, y_n + \frac{h}{3}k_1) \quad \text{and} \quad k_3 = f(x_n + \frac{2h}{3}, y_n + \frac{2h}{3}k_2)$$

   using the test equation $y' = \lambda y$. Give your answer for $h = 1/2, \lambda = -3$. **[5]**

   (c) Use Adams-Moulton 3rd order method

$$y_{j+1} = y_j + \frac{h}{12}(5f_{j+1} + 8f_j - f_{j-1})$$

   to compute $y(0.4)$ with $h = 0.2$ where y satisfies $y' = 2t + y^2, y(0) = 1$. Use any second order scheme to evaluate the past value required and do two iterations of Newton-Raphson's method to solve the non-linear algebraic equation. **[5]**

3. (a) By using Routh-Hurwitz criterion, find the condition on λh so that the linear multi-step method

$$y_{n+2} - y_n = \frac{h}{2}(f_{n+1} - 3f_n)$$

   for solving $y' = \lambda y$ is absolute stable. **[6]**

   (b) Consider the IVP $y' = x + y, y(0) = 0$. If

$$y_{n+1} = y_n + \frac{h}{2}(f_{n-2} - f_{n-1})$$

   is used for the Prediction and for the Correction method

$$y_{n+1} = y_n + h(f_{n+1} - f_{n-1})$$

   is used, find the value of $y(0.3)$ by using two iterations of Corrector formula with $h = 0.1$. **[5]**

END