BITS, PILANI - K K BIRLA GOA CAMPUS

FIRST SEMESTER 2019-2020

Comprehensive Exam (Closed Book)

Course Name : Nonlinear Optimization	**Date : 08/12/2019**
Course No. : MATH F471	**Time : 3 Hours**
Day : Sunday	**Max. Marks : 95**

Instructions:

1. All questions are compulsory.
2. Write all the steps clearly and **give explanation for complete credit**.
3. Start a new question on a fresh page.

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1. Convert the following separable programming problem to a linear programming problem **[10]**

$$\begin{aligned} \text{Min. } & f(x_1, x_2) = 4x_1^2 + x_2^2 + x_1 + x_2 \\ \text{subject to } & x_1^2 + x_2^2 \le 16, \\ & 2x_1 + 3x_2 \le 6, \\ & x_1, x_2 \ge 0. \end{aligned}$$

   Consider four equally spaced grid points for x_1 and five equally spaced grid points for x_2.

2. Consider the problem **[5 + 8]**

$$\begin{aligned} \text{Min. } & f(x_1, x_2) = \frac{x_1 + 3x_2 + 3}{2x_1 + x_2 + 6} \\ \text{subject to } & 2x_1 + x_2 \le 12, \\ & -x_1 + 2x_2 \le 4, \\ & x_1, x_2 \ge 0. \end{aligned}$$

   (a) Show that the KKT conditions are sufficient for the problem.

   (b) Show that any point on the line segment joining $(0, 0)$ and $(6, 0)$ is an optimal solution.

3. (a) Let $f(X)$ be a quadratic function with an $n \times n$ symmetric positive definite hessian matrix (H). Prove that if we successively take optimal steps along the H-conjugate directions $d_1, d_1, \cdots, d_n$, we will reach the optimal point in exactly n iterations.

   (b) Find the Min. $f(x_1, x_2) = x_1^2 - x_1x_2 + 3x_2^2$, starting with $X_1 = (1, 2)$ using the conjugate gradient method. **[8 + 12]**

4. Min. $f(x) = 4x^3 + x^2 - 7x + 14$ in the interval $[0, 1]$ within the interval of uncertainty $0.25L_0$, where L_0 is the initial interval of uncertainty, using Fibonacci search method. **[10]**

5. Prove or disprove the following statements: **[24]**

(i) Let S be a non empty convex set in $\mathbb{R}^n$, and let S_α be the level set of $f : S \to \mathbb{R}^n$. Then f is a convex function if and only if S_α is a convex set.

(ii) Let $d_1, d_2, \cdots, d_n$ be $H-$conjugate vectors, where H is an $n \times n$ symmetric positive definite matrix, then $d_1, d_2, \cdots, d_n$ are linearly independent vectors.

(iii) For the NLPP Min. $f(X) = x_1^2 + x_2$, subject to $x_1^2 + x_2^2 \leq 9, \quad x_1 + x_2 \leq 1$, the point $(0, -2)$ is a KKT-point.

(iv) Let $f_1, f_2, \cdots, f_m : \mathbb{R}^n \to \mathbb{R}$ be convex functions, then

$f(X) = \text{max.}\ \{f_1(X), f_2(X), \cdots, f_k(X)\}$ is a convex function.

6. Min. $f(x_1, x_2) = x_1 - 2x_2$, subject to, $-x_1 + x_2^2 \leq 1,\ x_2 \geq 0$, using the interior penalty function method by considering the log-barrier function and by taking the following values of penalty parameter $\mu = 1, 0.5, 0.1$, and 0.01. Hence write the optimal solution. **[10]**

7. State/Define the following: **[8]**

(a) The Caratheodory's Theorem.

(b) Supporting hyperplane for a set.

(c) Quasiconvex functions.

(d) Fitz John point for a NLPP.

******************************The End************************************