Birla Institute of Technology and Science, Pilani 333031

II Semester 2022-23

MATH F471 (NON LINIEAR OPTIMIZATION)

COMPREHENSIVE EXAMINATION

PART A : CLOSE BOOK

MAX. MARKS: 18 **MAX. TIME: 70 MINS** **DATE: MAY 20, 2023 (SATURDAY)**

Note: Use text book notations only.

Q.A1 a) Show that a closed half space in $\mathbb{R}^n$ is a convex set.

b) Is point (1.5, 6) is a convex linear combination of points (0, 0), (2, 0) and (1, 1)? If yes, then express it. **[4 + 5]**

Q.A2 Prove or disprove that the function $f(x_1, x_2) = x_1^4 - x_1x_2 + x_2^4 + 12x_1^2 + 6x_2^2 - x_1 - x_2$ is convex.

[4]

Q.A3 Consider the following problem

Maximize $x_1^2 + 2x_1x_2 + x_2^2$

subject to $x_1^2 + x_2^2 = 1$

a) Write KKT optimality conditions.

b) Find the point(s) satisfying the KKT conditions, verify whether or not the KKT point(s) are optimal point(s). **[5]**

*******END of PART A*******

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PART B (Open Book)

MAX. MARKS: 27 **MAX. TIME: 110 MINS** **DATE: MAY 20, 2023 (SATURDAY)**

Q.B1 Solve the following fractional programming problem by using Charnes and Cooper method (use M method to solve LPP)

$$\begin{aligned} &minimize\ \frac{-5x_1 - 2x_2 - 4}{4x_1 + 2x_2 + 5} \\ &\text{subject to } 3x_1 + 2x_2 \leq 6 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

[8]

Q.B2 Solve the following problem by using separable programming (consider grid points as 0, 1, 2, 3)

$$\begin{aligned} &maz\ f(x_1, x_2) = 3x_1^2 + 2x_2^2 \\ &\text{subject to } x_1^2 + x_2^2 \leq 9 \\ &\qquad x_1 + x_2 \leq 3 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

[8]

Q.B3 Find the x and $f(x)$ after **two** iteration for minimize $f(x_1, x_2) = 6x_1 - 3x_2^2 - x_1^2$, using the steepest decent starting with $X_0 = (1, 1)$. [7]

Q.B4 Consider the following problem:

$$Minimize\ z = x_1 - x_2 - 2x_3$$

$$Subject\ to\ x_1 + x_2 + x_3 \leq 5;\ \ x_1^2 + x_2^2 = 4; \qquad (x_1, x_2, x_3) \in \mathbb{R}^3$$

Find the Lagrangian Dual Function θ explicitly. **[4]**

****END ****