

Birla Institute of Technology and Science, Pilani 333031

II Semester 2022-23

MATH F471 (NON LINEAR OPTIMIZATION)

MID SEMESTER EXAMINATION (CLOSED BOOK)

MAX. MARKS: 35

MAX. TIME: 90 MINS

DATE: MARCH 18, 2023 (SATURDAY)

Note: Use text book notations only.

Q. 1 Define with example (i) Supporting hyperplane, (ii) quasi convex function. [4]

Q. 2 Let C be a nonempty closed convex cone. then $C = C^{**}$, where C^* represent polar cone of set C . [5]

Q. 3. Identify the closure, interior and boundary points of set

$$S = \{\mathbf{x} : 1 \leq x_1 \leq 2, x_2 = 3\}. \quad [3]$$

Q.4 State and proof Farka's theorem. [6]

Q.5 Let $\mathbf{g}: \mathbb{R}^m \rightarrow \mathbb{R}$ be a convex function, let $\mathbf{h}: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be an affine function of the form $\mathbf{h}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{b}$ where \mathbf{A} is $m \times n$ matrix and \mathbf{b} is an $m \times 1$ vector. Then show the composite function $f: \mathbb{R}^n \rightarrow \mathbb{R}$ defined as $f(\mathbf{x}) = g[\mathbf{h}(\mathbf{x})]$ is convex function. [6]

Q.6 Prove or disprove convexity or pseudo convexity of function

$$f(x, y) = \frac{x^2 + x + y^2}{x + 1}, \quad x + 1 > 0 \quad [5]$$

Q.7 Consider a problem

$$\text{Minimize } 6x_1^2 + 2x_2^2 - x_1x_2 - x_1 - x_2.$$

Write the first-order necessary optimality conditions. Is this condition also sufficient for optimality? Why? Also, find the minima if exist. [6]

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