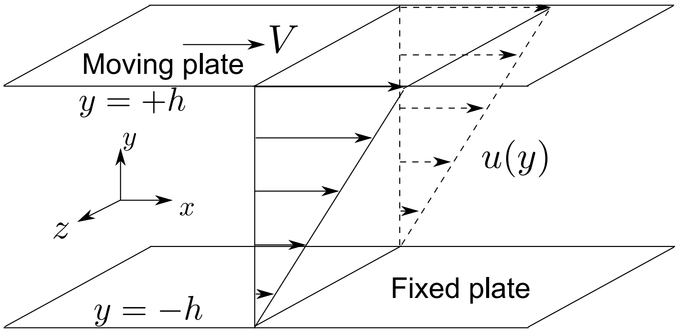
Birla Institute of Technology and Science Pilani
Department of Mechanical Engineering
Fluid Mechanics (ME F212)
Mid sem. Exam., Date: Oct. 31, 2022

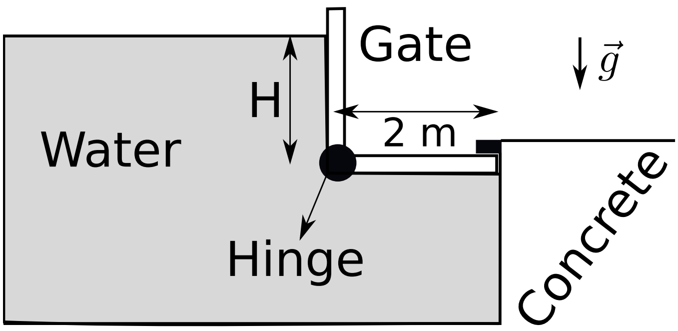
Timing: 2:00 to 3:30 PM | First sem. 2022-23 | Max mark: 50

Write your name and ID at the top clearly. Partial marks will be considered only when the exam. is attempted systematically and neatly. Take $\rho = 1000$ kg/m^3, $\mu = 0.001$ Pa·s, $\vec{g} = -9.81\,\hat{j}$ m/s^2 unless mentioned otherwise.

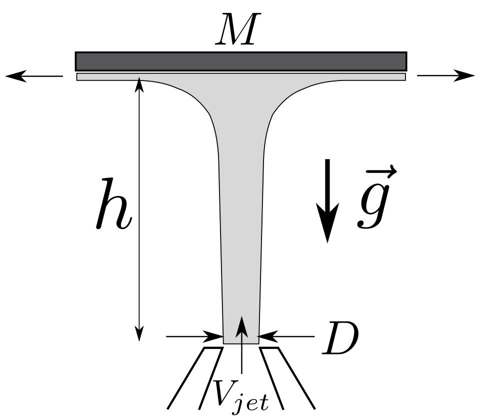
1. The velocity profile for a fluid of viscosity μ flowing between two parallel plates shown in the figure below is given by $\vec{u} = (\frac{V}{2h}y + \frac{V}{2})\hat{i}$. Calculate the following:
   (a) The viscous stress tensor in matrix form. (2 marks)
   (b) The total viscous force acting on the lower fixed plate of area A. (2 marks)
   (c) The magnitude and direction of rotation of the fluid elements. (2 marks)



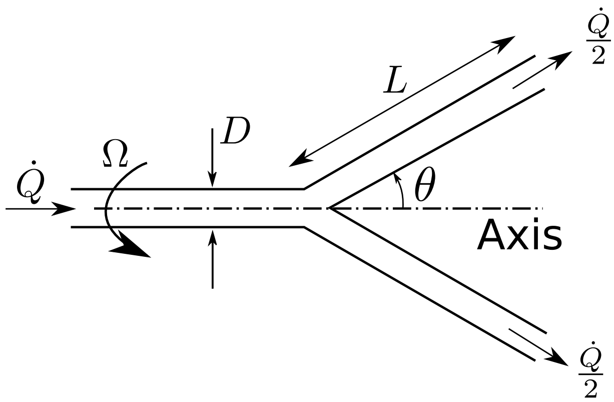
2. An 'L' shaped, mass less gate holds water in a lake as shown in the figure below. The gate is so designed that it will automatically open as the water level rises. Gravity acts downwards as shown in the figure. At what height 'H' above the hinge the water should rise so that the gate opens? (10 marks)



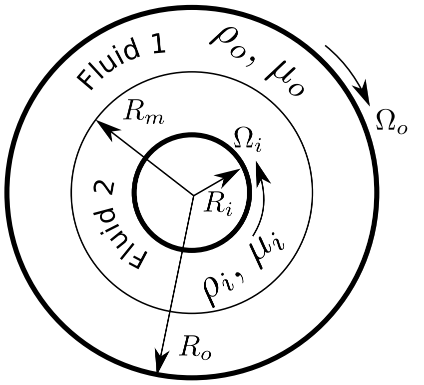
3. A 1.8 kg solid, circular disk is constrained to move horizontally but is free to move vertically as shown in the figure on the next page. The disk is struck from below by a circular jet of water which holds the disk stationary at a height h from the nozzle outlet. The speed of the water jet at the nozzle outlet is $V_{jet} = 12$ m/s and the jet diameter at the same location is $D = 30$ mm. Gravity acts downwards as shown in the figure. Based on this data, find the height h to which the disk will rise and remain stationary. You may have to use Bernoulli's equation somewhere. Clearly state any assumptions you are taking during the analysis. (10 marks)



4. A circular pipe of diameter D carrying water at a flow rate of $\dot{Q}$ bifurcates symmetrically into two legs of length L as shown in the figure below. The legs are inclined at an angle of θ with respect to the axis and their length L is much larger than the diameter D. The assembly rotates steadily at an angular velocity of Ω about the axis. There is no gravity in this case. Derive an expression of the torque required to rotate the assembly about the axis. Clearly state any assumptions you are taking during the analysis. (10 marks)



5. Two infinitely long, concentric, circular cylinders of radius R_i and R_o rotate at constant angular velocities Ω_i and Ω_o in opposite directions as shown in the cross-sectional view in the next figure. Two viscous fluids of properties ρ_i, μ_i and ρ_o, μ_o are kept inside the cylinders as shown. The interface between the fluids has radius R_m. There is no gravity in this case. Calculate the velocity distribution inside the two fluids under steady state condition. What are the viscous stresses in the two fluids. What is the pressure at the interface, p_{int}, and outer cylinder, p_o, if it is equal to p_i on the inner cylinder? Clearly state any assumptions you are taking during the analysis. (14 marks)



$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \tag{1}$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right\} + \rho g_r \tag{2}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right\} + \rho g_\theta \tag{3}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\} + \rho g_z \tag{4}$$

$$D_{rr} = \frac{\partial v_r}{\partial r},\ D_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r},\ D_{zz} = \frac{\partial v_z}{\partial z} \quad , \quad D_{r\theta} = \frac{r}{2}\frac{\partial (v_\theta / r)}{\partial r} + \frac{1}{2r}\frac{\partial v_r}{\partial \theta},\ D_{\theta z} = \frac{1}{2r}\frac{\partial v_z}{\partial \theta} + \frac{1}{2}\frac{\partial v_\theta}{\partial z},\ D_{zr} = \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \tag{5}$$

$$\tag{6}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = 0 \tag{7}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho g_x \tag{8}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho g_y \tag{9}$$

$$D_{xx} = \frac{\partial u}{\partial x},\ D_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad , \quad D_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\ D_{yy} = \frac{\partial v}{\partial y} \tag{10}$$

$$\tag{11}$$

$$\frac{\partial}{\partial t}\int_{cv} \rho \, dV + \int_{cs} \hat{n}\cdot\rho\vec{U}\, dS = 0 \tag{12}$$

$$\frac{\partial}{\partial t}\int_{cv} \rho\vec{U} \, dV + \int_{cs} \hat{n}\cdot\rho\vec{U}\vec{U}\, dS = -\int_{cs} p\hat{n}\, dS + \int_{cs} \hat{n}\cdot\tau\, dS + \int_{cv} \rho\vec{g}\, dV + \vec{F}_{ext} \tag{13}$$

$$\frac{\partial}{\partial t}\int_{cv} \rho(\vec{r}\times\vec{U}) \, dV + \int_{cs} \hat{n}\cdot\rho\vec{U}(\vec{r}\times\vec{U})\, dS = -\int_{cs} p(\vec{r}\times\hat{n})\, dS + \int_{cs} \vec{r}\times(\hat{n}\cdot\tau)\, dS + \int_{cv} \rho(\vec{r}\times\vec{g})\, dV + \vec{M}_{ext} \tag{14}$$