DATE: 26/12/2022

## Instructions:

- Write all steps while answering the problems.
- All the parts of a question must be answered together at a single place.
- Support your answers with appropriate sketches or free-body diagrams.
- Predefine all the symbols used.

1. A spherical thrust bearing is shown in figure. The gap between the spherical member and the housing is of constant width $h$. Obtain an algebraic expression for the torque on the spherical member, as a function of angle $\alpha$.

2. The 6 m long gate as shown in figure is a quarter circle and is hinged at H . Compute the horizontal and vertical components of the hydrostatic force on the gate and their line of action. Also, determine the horizontal force, P required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.
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3. Let the vortex-sink flow to simulate a tornado as in figure. Suppose that the circulation about the tornado is $\Gamma=8500 \mathrm{~m}^{2} / \mathrm{s}$ and that the pressure at $r=40 \mathrm{~m}$ is 2200 Pa less than the far-field pressure. Assuming inviscid flow at sea-level density, estimate (a) the appropriate sink strength $-m$, (b) the pressure at $r=15 \mathrm{~m}$, and (c) the angle $\beta$ at which the streamlines cross the circle at $r=40 \mathrm{~m}$. Take $\rho_{\text {air }}=1.225$ $\mathrm{kg} / \mathrm{m}^{3}$.

4. Consider water at $20^{\circ} \mathrm{C}$ flowing at $6 \mathrm{~m} / \mathrm{s}$ past a $1-\mathrm{m}$ diameter cylinder. What doublet strength in $\mathrm{m}^{3} / \mathrm{s}$ is required to simulate this flow? If the free stream pressure is 200 kPa , use inviscid theory to estimate the surface pressure at $180^{\circ}$. Suppose that circulation is added to the cylinder flow which is sufficient to place the stagnation points at $\theta=35^{\circ}$ and $145^{\circ}$. What is the required vortex strength in $\mathrm{m}^{2} / \mathrm{s}$ ? Compute the resulting pressure at the stagnation points.
5. A rectangular block of mass $\boldsymbol{M}$, with vertical faces, rolls without resistance along a smooth horizontal plane as shown. The block travels initially at speed $\boldsymbol{U}_{\boldsymbol{o}}$. At $\boldsymbol{t}=0$ the block is struck by a liquid jet and its speed begins to slow. Obtain an algebraic expression for the acceleration of the block for $t>0$ (write the governing equation and the assumptions involved). Solve the equation to determine the time at which $\boldsymbol{U}=0$.

6. A simplified sketch of a hydraulic turbine runner is shown in figure. Relative to the rotating runner, water enters at section (1) (cylindrical cross-section area $A_{l}$ at $r_{l}=1.5 \mathrm{~m}$ ) at an angle of $100^{\circ}$ from the tangential direction and leaves at section (2) (cylindrical cross-section area $A_{2}$ at $r_{2}=0.85 \mathrm{~m}$ ) at an angle of $50^{\circ}$ from the tangential direction. The blade height at section (1) and (2) is 0.45 m and the volume flow rate through the turbine is $30 \mathrm{~m}^{3} / \mathrm{s}$. The runner speed is 130 rpm in the direction shown. Determine the shaft power developed. Draw the velocity triangle for the flow at inlet and outlet of the turbine runner.

7. A viscous, incompressible fluid flows between the two infinite, vertical, parallel plates as shown in figure. Determine, by use of the continuity and Navier-Stokes equations, an expression for the pressure gradient in the direction of flow. Express your answer in terms of the mean velocity. Assume that the flow is laminar, steady, and fully developed. Also show how the velocity and pressure vary.

