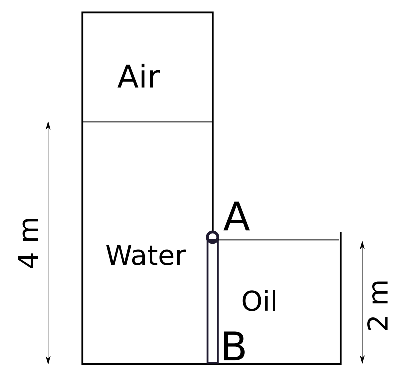
Name: **Id:**

Birla Institute of Technology and Science, Pilani
ME F212: Fluid Mechanics

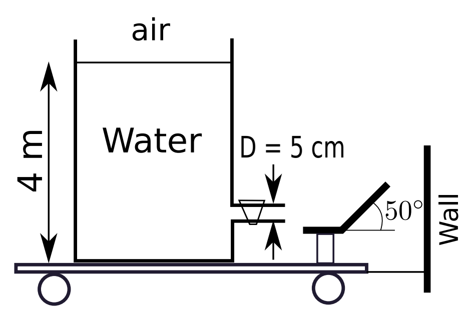
Comprehensive exam.: **Open Book** 90 Marks 09:00 AM-12:00 PM, 18/12/2023

Partial marking will be considered only when all the sub-steps are clearly written

1. A vertical gate AB is 2 meter high and 2 meter deep (in-paper depth) and is hinged at point A. The gas pressure above water is 20 kPa and atmospheric pressure of 101 kPa above the oil. The density of water is 1000 kg/m^3 and that of oil is 750 kg/m^3. What horizontal force (magnitude and direction) is required at point B for the equilibrium of the gate? (12 marks)



2. Water is initially at a height of 4 meter in a tank as shown in the figure below. The tank stands on a frictionless cart which is tied to the wall with the help of a rope. The valve at the side is opened so that tank feeds a jet of diameter 5 cm which is deflected by 50 degree by a vane as shown in the figure. Assuming frictionless flow, compute the tension in the rope at the start of the process. (11 marks)



3. A long, solid, horizontal circular cylinder of outer radius R_1 is placed concentrically inside a circular pipe of inner radius R_2. The annular region between these two is filled with a viscous fluid of viscosity μ and density ρ. A pressure gradient per unit length of $\frac{\Delta P}{L}$ is applied across the pipe ends to move the fluid. Considering steady, incompressible, axisymmetric, fully developed, laminar flow, calculate the maximum velocity inside the pipe and its radial location from the axis in terms of R_1 and R_2. (12 marks)

4. A 1/40 model of a ship with 1000 m^2 wetted area is towed in water at 2.5 m/s. The model experiences a fluid resistance of 25 N. Using concepts from modeling and similitude, calculate the following: (a) The corresponding speed of actual ship, (b) The wave drag on the ship, (c) the viscous drag on the ship

if the viscous drag coefficient for the model is 0.005 and for the actual ship is 0.02, (d) the total drag force on the ship, (e) the power required to propel the ship. (12 marks)

5. The equation governing the motion of a gas bubble of diameter D inside a liquid is the Navier-Stokes equation in vector form: $\rho\left(\frac{\partial \vec{U}}{\partial t} + \vec{U}\cdot\nabla\vec{U}\right) = -\nabla P + \mu\nabla^2\vec{U} + \rho\vec{g} + \sigma\kappa\hat{n}\delta_s$. In this equation, ρ is density, μ is viscosity, σ is the surface tension coefficient, κ is the curvature, δ_s is the surface delta function which implies that the surface tension force acts only at the interface between the gas and liquid. Assuming appropriate reference scales for the different quantities, non-dimensionalize the above equation. For your information, the surface tension waves move over the interface at a speed of $\sqrt{\frac{\sigma}{\rho D}}$. To non-dimensionalize pressure, use surface tension forces based pressure scale. (11 marks)

6. A 2D Rankine oval is formed by superimposing a source and a sink which are placed symmetrically about the origin and are 2 meter apart on the real axis. Both of these have strength equal to 10 m^2/s. A uniform flow parallel to real axis with a strength of 5 m/s is also there. Pressure far from the oval is P_{atm} = 101 kPa. Answer the following questions: (2 marks each)

   (a) What are the coordinates of the stagnation point?

   (b) What is the equation for the boundary of the oval?

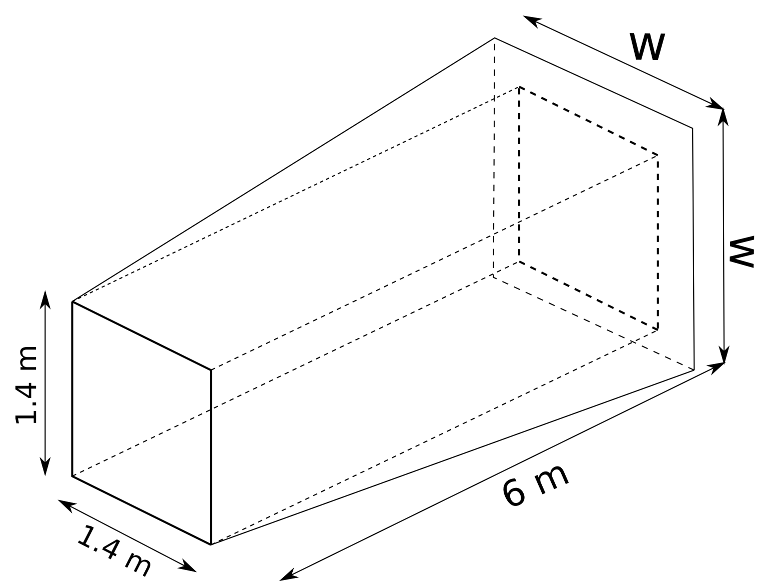
   (c) What is the width of the oval?

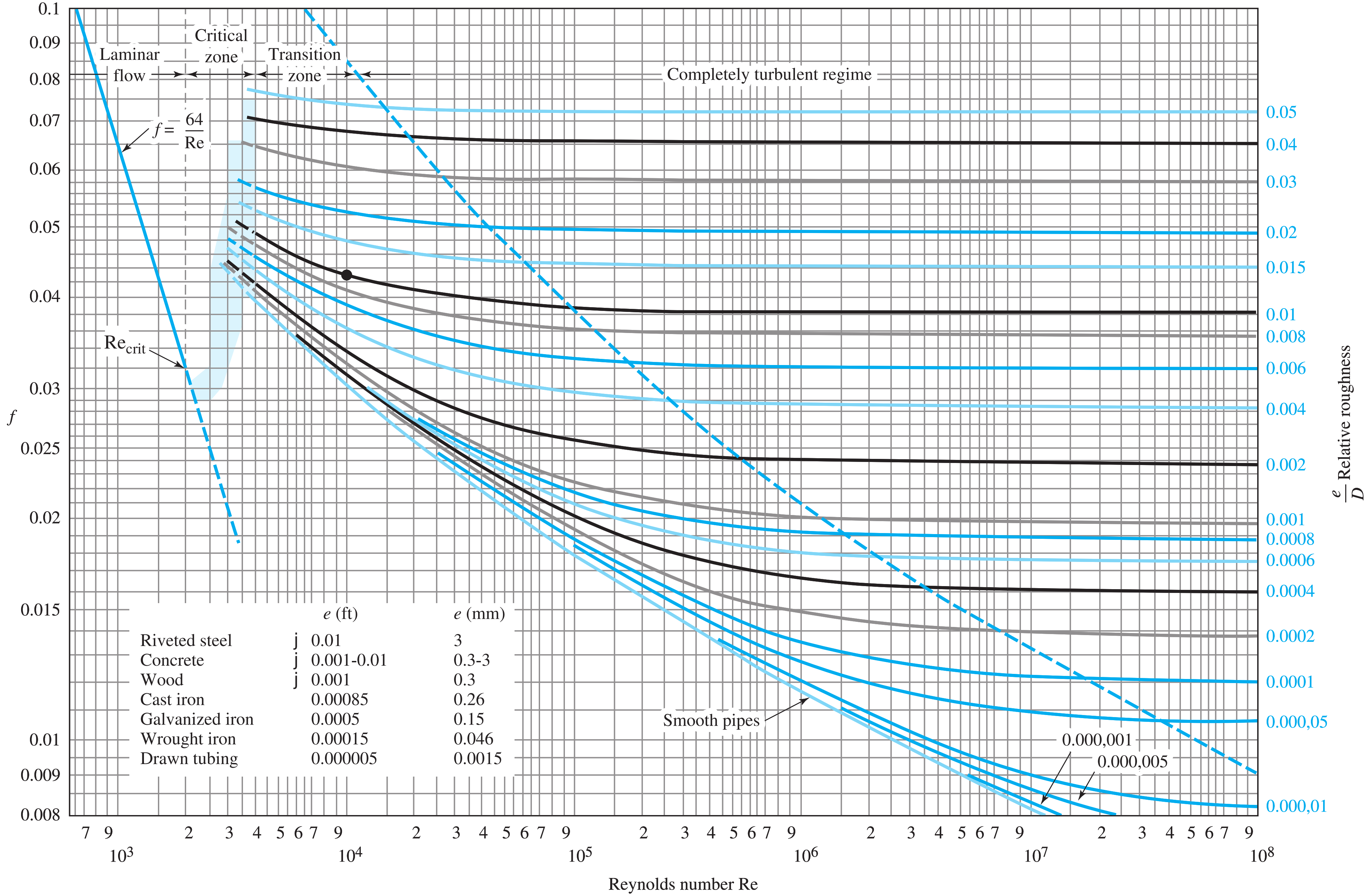
   (d) What is the pressure at the leading stagnation point?

   (e) What is the pressure at the half width location on the imaginary axis?

7. A galavanized steel pipe is required to carry water at 20°C with a velocity of 3.1 m/s. If the pressure drop over its 210 m horizontal length is not to exceed 15.5kPa, determine the required diameter of the pipe using the Moody's chart. (11 marks)

8. High speed air flow at 30° C with velocity equal to 60 m/s is desired throughout a wind tunnel of square cross-section with dimensions 1.4 m by 1.4 m as shown in the figure below. The inlet velocity is also 60 m/s. What should be the width W of the exit section which is at a distance of 6 meter from the inlet? (11 marks)





Some relevant equations and formulas are as follows:

- $\frac{\partial}{\partial t}\int_v \rho\vec{U}dV + \int_s \rho\vec{U}\vec{U}\cdot\hat{n}dS = -\int_s p\hat{n}dS + \int_s \tau\cdot\hat{n}dS + \int_v \rho\vec{g}dV + \vec{F}_{ext}$
- $\frac{\partial}{\partial t}\int_v \rho(\vec{r}\times\vec{U})dV + \int_s \rho(\vec{r}\times\vec{U})\vec{U}\cdot\hat{n}dS = -\int_s \vec{r}\times p\hat{n}dS + \int_s \vec{r}\times(\tau\cdot\hat{n})dS + \int_v \rho(\vec{r}\times\vec{g})dV + M_{ext}$

- $\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z} = 0$
- $\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial^2 v_r}{\partial z^2}\right\} + \rho g_r$
- $\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left\{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right\} + \rho g_\theta$
- $\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\} + \rho g_z$

- $D_{rr} = \frac{\partial v_r}{\partial r}$
- $D_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}$
- $D_{zz} = \frac{\partial v_z}{\partial z}$

- $D_{r\theta} = \frac{r}{2}\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{2r}\frac{\partial v_r}{\partial\theta}$
- $D_{\theta z} = \frac{1}{2r}\frac{\partial v_z}{\partial\theta} + \frac{1}{2}\frac{\partial v_\theta}{\partial z}$
- $D_{zr} = \frac{1}{2}\frac{\partial v_r}{\partial z} + \frac{1}{2}\frac{\partial v_z}{\partial r}$

- $\omega_r = \frac{1}{r}\frac{\partial v_z}{\partial\theta} - \frac{\partial v_\theta}{\partial z}$
- $\omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r}$
- $\omega_z = \frac{1}{r}\frac{\partial(rv_\theta)}{\partial r} - \frac{1}{r}\frac{\partial v_r}{\partial\theta}$

- $I = \frac{BH^3}{12}$
- $I = \frac{\pi R^4}{4}$
- $I = \frac{\pi ab^3}{4}$