BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2022-23
ME F312: Adv Mechanics of Solids
Mid-Semester Exam (OPEN BOOK)

Max Marks: 30 Duration: 1.5 Hours (2PM -3:30PM) Date: November 2, 2022

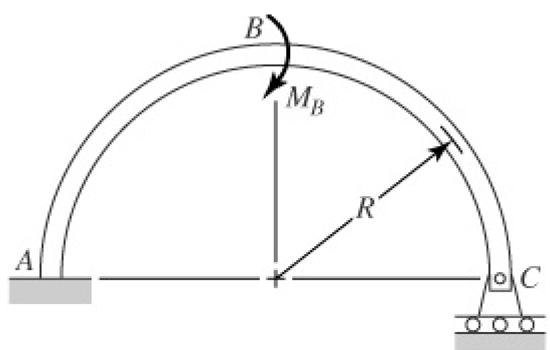
Q1. An elastic layer is sandwiched between two perfectly rigid plates, to which it is bonded. The layer is compressed between the plates, the direct stress being σ_z. Supposing that the attachment to the plates prevents lateral strain ε_x, ε_y completely, find the apparent Young's modulus (that is, $\frac{\sigma_z}{\varepsilon_z}$) in terms of E and υ. **[4]**

Q2. The stress array relative to axes (x, y, z) is given by: $T = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 6 & 0 \\ 2 & 0 & 8 \end{bmatrix}$ MPa.

(a) Determine the stress invariants of T. **(b)** Out of these three stress invariants, which one is responsible for failure of a ductile material and why? Give justification of your answer.
(c) compute the principal stresses and octahedral shear stress. **[6]**

Q3. The displacement field is given by: $u_x = k(x^2 + 2z)$, $u_y = k(4x + 2y^2 + z)$, $u_z = 4kz^2$ where k is a constant small enough **(**$k = 10^{-3}$**)** to ensure applicability of the small deformation theory.
Determine **[10]**
(a) the strain matrix at point **P** (3, 2, 4)
(b) the principal strains and the direction cosines associated with minimum principal strain
(c) the strain along **PQ**: $n_{x2} = 0, n_{y2} = 1/\sqrt{2} = n_{z2}$

Q4. Member ABC in Fig has a uniform circular cross-section with radius r that is small compared with R.
Determine **(a)** the pin reaction at C **(b)** the horizontal component of the displacement of point B and **(c)** the change in slope of the cross section at point B for the member. **[10]**



Q1.

Q2.

$$\begin{bmatrix} \varepsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \varepsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \varepsilon_{zz} \end{bmatrix} = \text{ at point } (2, 2, 3) \qquad \begin{bmatrix} 4 & 2 & \frac{1}{2} \\ 2 & 8 & 1 \\ \frac{1}{2} & 1 & 24 \end{bmatrix} k$$ **[1M]**

$$\varepsilon^3 - J_1\varepsilon^2 + J_2\varepsilon - J_3 = 0$$

<u>Volumetric strain</u> $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_x + \varepsilon_y + \varepsilon_z = 36\times10^{-3}$ **[1M]**

$\varepsilon_1 - \bar{\varepsilon} = 12.08\times10^{-3}$

<u>Deviatoric strain components</u> $\varepsilon_2 - \bar{\varepsilon} = -8.83\times10^{-3}$ **[1M]**

$\varepsilon_3 - \bar{\varepsilon} = -3.25\times10^{-3}$

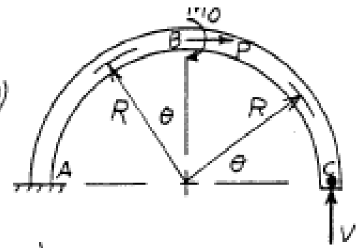
$$\varepsilon_{pq} = \varepsilon_{xx}n_x^2 + \varepsilon_{yy}n_y^2 + \varepsilon_{zz}n_z^2 + \gamma_{xy}n_x n_y + \gamma_{yz}n_y n_z + \gamma_{zx}n_z n_x \Rightarrow \varepsilon_{pq} = 17.76k$$ **[2M]**

$$\cos\theta = n_{x1}n_{x2} + n_{y_1}n_{y_2} + n_{z1}n_{z2}$$ **[1M]**

$$\cos\theta' = \frac{1}{(1+\varepsilon_{pq})(1+\varepsilon_{pr})}\begin{bmatrix} 2\varepsilon_{xx}n_{x1}n_{x2} + 2\varepsilon_{yy}n_{y_1}n_{y_2} + 2\varepsilon_{zz}n_{z1}n_{z2} \\ +\gamma_{xy}(n_{x1}n_{y2} + n_{x2}n_{y1}) + \gamma_{yz}(n_{y1}n_{z2} + n_{y2}n_{z1}) \\ +\gamma_{zx}(n_{x1}n_{z2} + n_{x2}n_{z1}) \end{bmatrix}$$

$\theta' - \theta = 55.5 - 54.82 = 0.68$ **[2M]**

Q3.



5.72 Let P be infinitesimal.

From C to B — $M = VR(1-\cos\theta)$

From B to A — $M = VR(1+\sin\theta) - M_0 - PR(1-\cos\theta)$

$$q_V = 0 = \int_0^{\pi/2} \frac{M}{EI}\frac{\partial M}{\partial V} R\,d\theta + \int_0^{\pi/2} \frac{M}{EI}\frac{\partial M}{\partial V} R\,d\theta$$

$$= \int_0^{\pi/2} \frac{VR(1-\cos\theta)}{EI}\left[R(1-\cos\theta)\right] R\,d\theta + \int_0^{\pi/2} \frac{VR(1+\sin\theta)-M_0}{EI}\left[R(1+\sin\theta)\right] R\,d\theta$$

$$0 = \frac{\pi VR^3}{2} - 2VR^3 + \frac{\pi VR^3}{4} + \frac{\pi VR^3}{4} + 2VR^3 + \frac{\pi VR^3}{4} - \frac{\pi M_0 R^2}{2} - M_0 R^2$$

$$V = \frac{M_0(\pi+2)}{3\pi R} = \underline{0.5455\,\frac{M_0}{R}}$$

$$q_P = \int_0^{\pi/2} \frac{VR(1+\sin\theta)-M_0}{EI}\left[-R(1-\cos\theta)\right] R\,d\theta$$

$$= -\frac{VR^3}{EI}\left(\theta - \cos\theta - \sin\theta - \frac{\sin^2\theta}{2}\right)\Big|_0^{\pi/2} + \frac{M_0R^2}{EI}\left(\theta - \sin\theta\right)\Big|_0^{\pi/2}$$

$$= -\frac{VR^3}{EI}\left(\frac{\pi}{2} + 1 - 1 - \frac{1}{2}\right) + \frac{M_0R^2}{EI}\left(\frac{\pi}{2} - 1\right)$$

$$= \underline{\frac{M_0R^2}{6\pi EI}\left(2\pi^2 - 7\pi + 2\right)}$$