Birla Institute of Technology & Science, Pilani
First Semester 2022-2023
Mid-Semester Examination

Course No.	: ME F342
Course Title	: COMPUTER AIDED DESIGN
Nature of Exam	: Open Book
Weightage	: 30% (60 Marks)
Duration	: 1.5 Hours
Date of Exam	:

No. of Pages = 1
No. of Questions = 4

Note:
1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q.1. The coordinates of a triangular object **ABC** are given by: **A** (0, 0, 0), **B** (0, 1, 0) and **C** (0, 0, 1). The coordinates of a rectangular object are given by **P** (3, 3, 0), **Q** (5, 3, 0), **R** (5, 5, 0) and **S** (3, 5, 0). Determine the transformation matrix to transform **ABC** without changing its shape so that point **A** coincides with point **S** and the line **AC** lies on line joining **Q** and **S**. **[12]**

Q.2. **(a)** Consider a parametric curve represented in the following form: **r**(t) = $\mathbf{P}_1\psi_1(t)$ + $\mathbf{P}_2\,\psi_2(t)$+ $\mathbf{P}_3\,\psi_3(t)$.

$\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ are any three control points in R^3. Three different types of basis functions are available:

(i) $\psi_i(t) = \left[\frac{t^2}{3t^2+2}, \frac{t^2-2t+1}{3t^2+2}, \frac{t^2+2t+1}{3t^2+2}\right]$ (ii) $\psi_i(t) = \left[\frac{t^2}{3t^2+2}, \frac{t^2-1}{3t^2+2}, \frac{t^2+1}{3t^2+2}\right]$

(iii) $\psi_i(t) = [\sin^2 t - t^2, \cos^2 t \quad t^2]$ $0 \le t \le 1$

For all basis functions, determine whether the shape of the curve is invariant or not under affine transformation (under translation and rotation) of the three control points. Also, determine the curves which are bounded by the triangle formed by $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$.

(b) A cubic Bezier curve $P(t)$ $(0 \le t \le 1)$ with defining polygon points $\boldsymbol{B_0}, \boldsymbol{B_1}, \boldsymbol{B_2}, \boldsymbol{B_3}$ is subdivided at $t = 1/3$ into two cubic Bezier curves, $Q(u)$ $(0 \le u \le 1,\ 0 \le t \le 1/3)$ and $R(v)$ $(0 \le v \le 1,\ 1/3 \le t \le 1)$. Determine defining polygon points $\boldsymbol{C_0}, \boldsymbol{C_1}, \boldsymbol{C_2}, \boldsymbol{C_3}$ for $Q(u)$.
[7 + 9 = 16]

Q.3. The defining polygon points are given by $\boldsymbol{B_1}[0,0]$, $\boldsymbol{B_2}[1,3]$, $\boldsymbol{B_3}[2,1]$ and $\boldsymbol{B_4}[3,2]$.
(a) Determine the tangent vectors for third order open uniform B-spline curve at starting and end points.
(b) Determine the tangent vectors for the periodic cubic B-spline curve at starting and end points.
(c) Find out the starting and end points of the cubic periodic B-spline curve whose tangent vectors at starting and end points are modified to [1, 1] and [1, –1], respectively using two pseudo vertices at start and end.

[4 + 4 + 12=20]

The recursive relation for B – spline basis functions is given by:

$$N_{i,k}(t) = \frac{t - x_i}{x_{i+k-1} - x_i} N_{i,k-1}(t) + \frac{x_{i+k} - t}{x_{i+k} - x_{i+1}} N_{i+1,k-1}(t)$$

Q.4. An ellipse $P(\theta)$ with semi-major axis a = 5 and semi minor axis b = 2 has center at (1, 1) and is inclined to the positive x-axis at 45^0. Find out the point on the sweep surface Q(θ, s) for $\theta = 90^0$ and s = 0.5 when the ellipse $P(\theta)$ is translated along z-axis by 10 units. $(0 \le s \le 1)$
[12]

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