

**Birla Institute of Technology and Science Pilani, K K Birla Goa Campus**  
**FIRST SEMESTER 2022-2023**

**ME F423 Microfluidics & its applications**  
**Comprehensive Exam (Closed Book)**

**DATE: 21/12/2022 Time: 2:00 P.M. – 05:00 PM Maximum Marks: 40**

---

**Instructions**

- a) Write all steps and mention all **assumptions** involved while answering the problems.
  - b) Answer each **main** question on a **new page**.
  - c) Use of **schematics** is encouraged
- 

**Q. 1.** Answer the following questions in **brief** with appropriate **reasoning** [2 x5= 10]

a) One of the class presentation subjects was “Inertial Microfluidics”. What is understood by this term *Inertial microfluidics*. Provide one example and application.

b) Droplets in the dripping regime are formed when order of capillary number (Ca) are small. When capillary number is larger than 1, droplets are in the jetting regime. Using the following data find the appropriate regime of the droplet. Given: Viscosity of inner fluid  $10^{-3}$  Pa-s, outer fluid  $10^{-2}$  Pa-s, inner fluid velocity 1 mm/sec; outer fluid velocity 10 mm/sec, and surface tension 40 mN/m, diameter of channel is 100  $\mu\text{m}$ .

c) What is understood by the term slip boundary condition? Explain. TMAC =1 represents perfect slip or no slip?

d) Provide two differences between flow at microscale and macroscale.

e) Provide two advantages of Di-electrophoresis. Represent the phenomenon of positive-Dielectrophoresis (p-DEP) using a schematic.

**Q.2** Consider gas flow between two parallel plates (Poiseuille flow) as shown in figure 1, the walls of the microchannel are separated by a distance  $h$ . With appropriate assumptions and using slip boundary conditions find the velocity profile in terms of Knudsen number. Consider 2nd order slip model to be applicable, refer to the general slip relation. Compare the velocity profile obtained above with the case when ‘no-slip’ conditions exist.

Now, find the slip velocity at the wall using following data  $\text{Kn}=0.1, \sigma= 0.8, L=0.5 \text{ m}, dP=0.01 \text{ N/m}^2, h =100 \mu\text{m}, \mu= 0.001 \text{ Pa-sec}$ . [10]

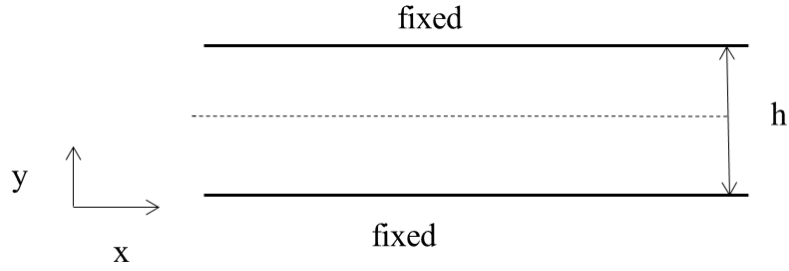


Figure 1

The general slip relation is given by

$$u_g - u_w = \frac{2 - \sigma}{\sigma} \left[ \lambda \left( \frac{\partial u}{\partial y} \right) + \frac{\lambda^2}{2} \left( \frac{\partial^2 u}{\partial y^2} \right) + \dots \right]$$

Q3. a) Explain the formation of an Electric Double layer (Use schematics and potential distribution diagram for the explanation). Using the Poisson's equation and the Boltzmann distribution obtain the Poisson-Boltzmann equation for a symmetric electrolyte. Using the P-B equation and 1D scaling analysis show the dependency of Debye length. Note the Boltzmann

distribution is given by:  $n = n_o e^{-\frac{ze\psi}{k_B T}}$  and Poisson's equation is given by  $\nabla^2 \psi = -\frac{\rho_e}{\epsilon}$  [8]

(b) Consider electro-osmosis as shown in the figure 2, the fluid flows in the microchannel under the influence of electrical field only. Write the momentum equation for this case under the assumption of low Re and fully developed flow conditions. Simplify the momentum equation obtained using the Poisson's equation. Find the expression for Helmholtz-Smoluchowski velocity scale. Using appropriate boundary conditions to determine an expression for the velocity profile. Draw the velocity profile. [4]

The Poisson's equation is given by:  $\nabla^2 \psi = -\frac{\rho_e}{\epsilon}$

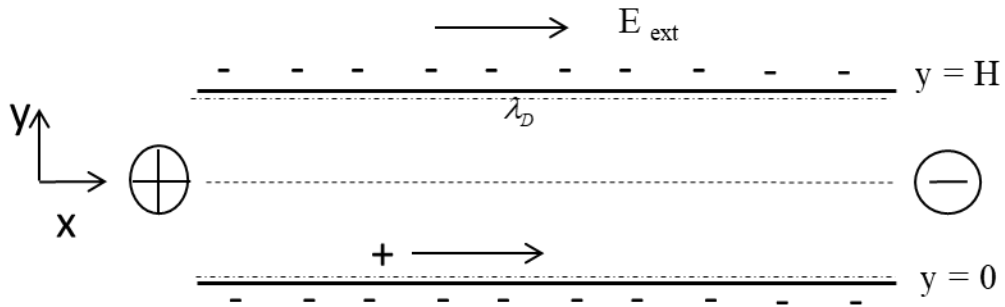


Figure 2

Q4. a) An electroosmotic flow takes place inside a microchannel of cross section  $20 \mu\text{m} \times 400 \mu\text{m}$  and  $2 \text{ cm}$  long under the applied potential of  $30 \text{ V}$ . The zeta potential of the wall is  $20 \text{ mV}$ . The viscosity of the liquid is  $1 \text{ m Pa}\cdot\text{sec}$ , and permittivity of the working fluid is  $45 \times 10^{-12} \text{ C/V}\cdot\text{m}$ . Find the electroosmotic flow rate.

b) Huckel's equation for electrophoresis can be obtained by equating the electrostatic forces with the Stokes viscous drag force. Find the expression for electrophoretic velocity of a particle. Consider debye length to be very large in comparison to the radius of the sphere. Note that the charge on the sphere is given by

$q_{sp} = \frac{4\pi\epsilon R^2}{\zeta} \left[ \frac{1}{\lambda} + \frac{1}{R} \right]$ . Find the electrophoretic velocity, if ionic mobility of the potassium particle is  $7 \times 10^{-8} \text{ m}^2/\text{s}\cdot\text{V}$  and applied electric field is  $10^4 \text{ V/m}$ .