**Birla Institute of Technology and Science Pilani, KK Birla Goa Campus**
**FIRST SEMESTER 2022-2023**
**ME F423 Microfluidics & its applications**
**Mid Semester Exam (Closed Book)**
**DATE: 02/11/2022 Time: 9:00 A.M. – 10:30 AM Maximum Marks: 30**

**Q1. Answer the following questions in brief with appropriate reasoning [9x 2=18]**

a) Consider the 1D advection-diffusion equation. Using order of magnitude analysis, find the ratio of advection time scale to diffusion time scale.

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}$$

b) Consider electro-wetting on a dielectric surface. How does the surface tension at the solid- liquid interface with the application of voltage during the actuated state. Does it increase or decrease as compared to the unactuated state. Also, is it possible to generate de-wetting of a drop?

c) Provide two applications of Microfluidics and mention any two commercialized Point-of care devices.

d) Using simple scaling analysis devise expressions for Capillary number and Weber number. Also find out a useful non-dimensional number by evaluating the ratio of Weber number to Capillary number.

e) Find the expression for Young's laplace pressure drop across a liquid gas interface with surface tension 'σ' inside a flat and very wide rectangular microchannel of height h', where the contact angle for both the top and bottom channels are same and equal to 'θ' as shown in fig. 1, assume h<<w.

f) Consider flow in the microchannel as shown in fig 2. Is it possible to obtain the Resistance ratio of the branches? If yes, provide this ratio in terms of flow rates.

g) Draw a simple schematic showing the process of dispersion. Which kind of velocity profile is suited to avoid transverse dispersion?

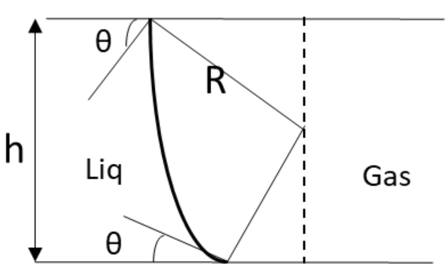


Fig 1

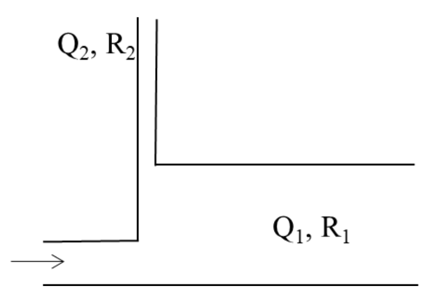


Fig 2

i) While deriving the expression of stokes drag, we found that the total drag on a spherical object is $F = 6\pi\mu Rv$. The total drag comprises of two drag components, the skin friction drag and the pressure drag. Evaluate the expression for skin friction drag.

$$dF_{d,skin} = \int_0^{\pi} \tau_{r\theta} \sin\theta . 2\pi R \sin\theta . R d\theta$$

$$where\ \tau_{r\theta} = \frac{3\mu u_{\infty} \sin^2\theta}{2R}$$

j) Consider a fully develop laminar flow in a microcapillary, show that the heat transfer rate is better in a microcapillary compared to its macro-counterpart.

Q2. (a) A H-shaped filter design is used to separate proteins from ribosomes at room temperature (25 $^{o}$C), shown in in Figure 3. Protein and ribosomes are spherical in shape with sizes 10 nm and 30 nm in diameter, respectively. The channel width is 100 µm. The average velocity of the flow is 1mm/sec. Find the range of length required in mm for successful operation of the H-filter. You may consider water as buffer with viscosity of 1cP and 1D diffusion model to be applicable with particles diffusion based on half width of the channel. **[4]**

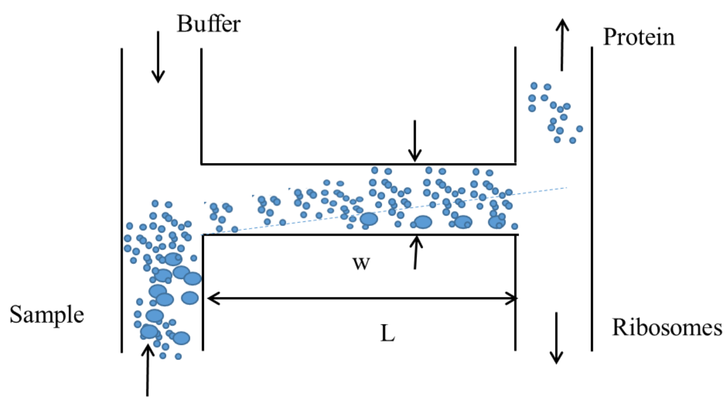


Figure 3: H-Filter design

Q3. Consider a 2D flow between two parallel plates (Poiseuille flow) as shown in figure 4, the walls of the microchannel are separated by a distance H. Assuming flow to be steady, Newtonian, incompressible, laminar and fully developed, find the expression for the velocity profile, determine ratio of average velocity by maximum velocity ratio and wall shear stress [Note: List your assumptions while simplifying the governing equations]. **[4]**

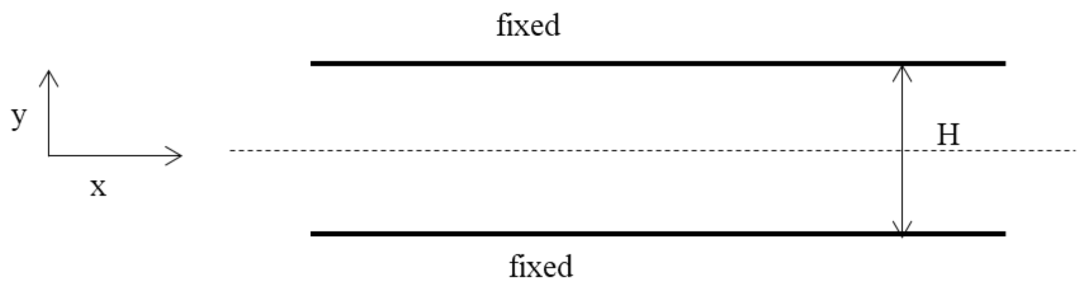


Figure 4

Q3. Consider a 5 mm long mercury droplet inside a microcapillary of 200 μm diameter, figure 5. The temperature, $T$, of the contact line at the heater end is equal to 35 °C, and room temperature of the other end is equal to 25 °C. Surface tension of mercury in air at 25 °C is equal to 486.5 mJ/m$^2$. The surface tension of mercury decreases with temperature as per the following relation $\sigma = 486.5 - 10\Delta T$ . **[4]**

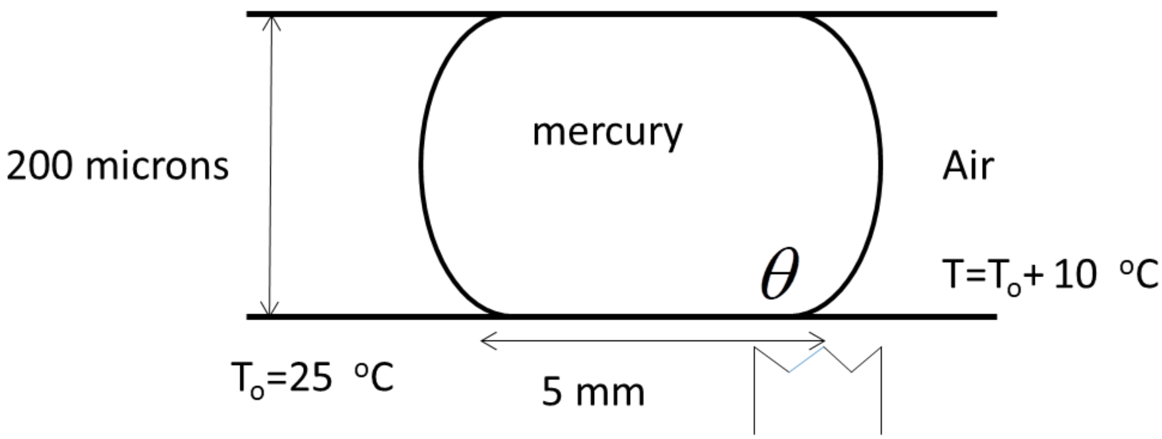


Figure 5

where $\Delta T$ is equal to ($T$ − 25) °C and $T$ is the temperature in degree Celsius. The static contact angle of mercury is equal to 140°. Density of mercury is 13,600 kg/m$^3$. Viscosity of mercury is equal to 1.526 × 10$^{-3}$ Pa-s. Show the direction of movement for the droplet and calculate the speed of the mercury droplet in meter per second.

------------------------------------------------END----------------------------------------------------------