Birla Institute of Technology and Science, Pilani
ME F485 – Numerical Methods in Fluid Flow and Heat Transfer
Mid-Semester Examination, Spring 2017-2018
(Closed Book)

Date:10/3/2018 **Duration: 9-10:30AM** **Marks [75]**

1. Multiple Choice Questions. (1M for each correct choice, -0.5 M for incorrect choice) [15M]

a. Select all that apply for TDMA.

i. Tridiagonal Solver
ii. Thomas Algorithm
iii. Indirect solution method
iv. $O(n)$ operations, where n is the size of the system
v. LU decomposition
vi. Gauss Elimination
vii. Direct Solver

b. For a certain 2D elliptic problem, Jacobi scheme leads to non-convergence. What options are at disposal to a numerical scientist to overcome convergence issues?

i. Gauss-Seidel iterative scheme
ii. Increase mesh refinement
iii. Under-relaxation
iv. Higher order finite difference schemes
v. Alternate Direct Implicit
vi. Employ Direct Solvers

c. Select the statements that apply to Multigrid Framework

i. More than one mesh refinements used
ii. Offers good compromise between speed and accuracy
iii. Free of truncation errors
iv. Prolongation is akin to interpolation operation
v. Achieves speed up by operating on coarser grids where spectral radius is smaller.
vi. Offers accuracy of finest grid in work units similar to the coarsest grid.

d. Select statements that pertain to hyperbolic equation,

i. Any point in solution space is only affected by its domain of disturbance
ii. Solution is a hyperbolic function
iii. Effect of disturbance travels at a finite wave speed along its characteristics directions.
iv. Unsteady heat conduction equation
v. Must have discontinuity in the solution

2. Classify the following equations into hyperbolic, elliptic, parabolic. [5M]

a. $\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0$

b. $\frac{\partial \phi}{\partial t} - \Gamma\frac{\partial^2 \phi}{\partial x^2} = 0$

c. $(1 - M^2)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

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3. Consider the following non-uniform grid. Derive a central finite difference scheme for $\left.\frac{\partial \phi}{\partial x}\right|_i$ as an average of forward difference and backward difference in terms of Δx_- and expansion ratio/growth ratio $g = \frac{\Delta x_+}{\Delta x_-}$. Show at least 2 terms in truncation error. What is the Leading Error Term(LET)? Is the scheme still second order? Comment on effect of g on truncation error.[15M]

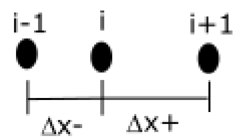
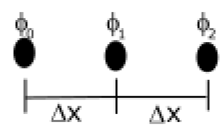


Figure Q3. Non-Uniform Finite Difference Grid

4. Consider integral form of governing equation over the domain Ω, $\int_\Omega \frac{d^2\phi}{dx^2} d\Omega = \int_\Omega S d\Omega$. Approximate piecewise polynomial solution of the form $\hat{\phi}(x) = ax^2 + bx + c$ is sought. Device second order forward Finite Difference scheme to express equation for left boundary node ϕ_0(Figure 2) having Neumann boundary condition prescribed, $\frac{d\phi}{dx} = f$. [15M]

Figure Q4. 1-Dimensional uniform finite difference grid



5. The gradient of pressure can be approximated by

(A) $\left.\frac{\partial p}{\partial x}\right|_i = \frac{p_{i+1} - p_{i-1}}{2\Delta x}$ or $(B) \left.\frac{\partial p}{\partial x}\right|_i = \frac{p_{i+1/2} - p_{i-1/2}}{\Delta x}$,

Which one do you think will be more accurate? Find the difference between the two in the form $(A) = (B) + a\Delta x^n \left.\frac{\partial^m p}{\partial x^m}\right|_i$. Specify a, n, and m. [15M]

$$Taylor\ Series: p(x+\Delta x) = p(x) + \Delta x\, p'(x) + \frac{\Delta x^2}{2} p''(x) + \frac{\Delta x^3}{6} p'''(x) + \frac{\Delta x^4}{24} p''''(x) + \cdots$$

6. What simplification can you make to the following generalized conservation equations for incompressible, inviscid, steady air flow over an airfoil? Justify your assumptions. [10M]

Continuity:
$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\left(\rho \vec{V}\right) = 0$$

Momentum Cons.:
$$\frac{\partial\left(\rho \vec{V}\right)}{\partial t} + \vec{\nabla}.\left(\rho \vec{V}\vec{V}\right) = -\vec{\nabla}p + \vec{\nabla}.\vec{\vec{\tau}} + \vec{B}$$

Energy Cons.:
$$\frac{\partial(\rho e)}{\partial t} + \vec{\nabla}.\left(\rho e \vec{V}\right) = -\frac{\partial p}{dt} - \vec{\nabla}.\left(p\vec{V}\right) + \vec{B}.\vec{V} + \vec{\nabla}.\left(\vec{\vec{\tau}}.\vec{V}\right) + \vec{\nabla}.\left(k\vec{\nabla}T\right) + \dot{q}$$