Birla Institute of Technology and Science, Pilani
ME F485: NFFHT

Comprehensive exam., **Open Book**, 40 Marks, 10:30-12:30 AM, 12/05/2023

- **Write neatly. Strike out rough work. Write your name and id. clearly.**
- **Partial marks will be awarded only when all the steps are clearly shown.**

1. Consider the following model equation: $a\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}$. Let $K = \nu/a$. Do the following:

   (a) Write an explicit formulation using a first order differencing in x and a second order central differencing in y. (2 marks)

   (b) Use the von Neumann stability analysis to determine the stability requirements of the scheme. (4 marks)

2. For the first order hyperbolic equation $\frac{\partial \phi}{\partial t} + a\frac{\partial \phi}{\partial x} = 0$, the following numerical scheme is proposed:

   $\phi_i^{n+1} = \frac{\phi_{i+1}^n + \phi_{i-1}^n}{2} - \frac{a\Delta t}{2\Delta x}(\phi_{i+1}^n - \phi_{i-1}^n)$.

   Determine the artificial viscosity introduced by this scheme. (5 marks)

3. Consider the following second order differential equation: $\frac{\partial^2 \phi}{\partial x^2} = e^{-x}$ and the boundary conditions: $\phi(0) = 0$ and $\phi(1) = 1$. Answer the following questions:

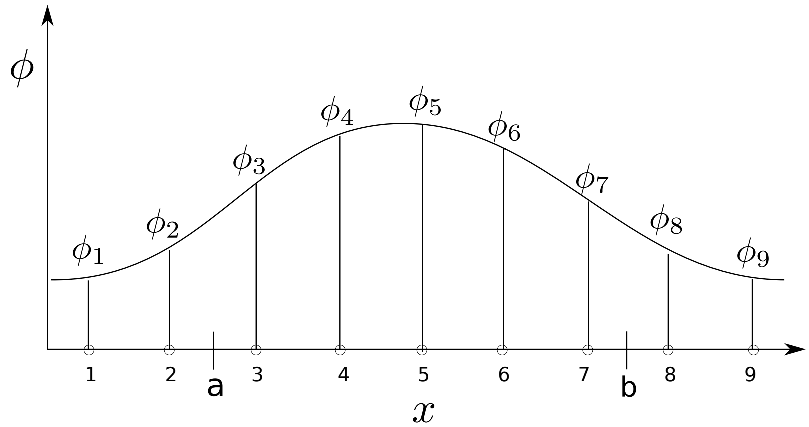
   (a) Solve the equation analytically to determine $\phi(x)$ and plot $\phi(x)$ versus x. (2 marks)

   (b) Discretize the equation using second order accurate finite difference method. Use a total of five equispaced nodes (including boundary nodes) and calculate the value of ϕ at the interior nodes. (3 marks)

   (c) What is the percentage error in the finite difference based value of ϕ at the interior nodes. The results should have four significant digits after the decimal point. (2 marks)

4. You want to solve the Laplace equation $\nabla^2\psi = 0$ numerically on a square domain of side length 4 m using finite volume method. Following boundary conditions are given: $\psi_{\text{Left}} = 10$, $\psi_{\text{Top}} = 5$, $\psi_{\text{Bottom}} = 1$, $\frac{\partial \psi}{\partial x}|_{\text{Right}} = 0$. Using four square grid cells each with side length equal to 1 m, calculate the value of ψ at the center of the grid cells. (5 marks)

5. For the distribution of a variable ϕ in the one dimensional domain shown in the figure on the next page, what is the value of ϕ at control volume faces at locations a and b? The grid size is 0.1 m and flow is from right to left with a velocity of -1 m/s. The fluid density is $\rho = 1$ kg/m^3 and coefficient of diffusion $\Gamma = 1$ kg/m·s. $\phi_1 = 1$, $\phi_2 = 1.4$, $\phi_3 = 2$, $\phi_4 = 2.3$, $\phi_5 = 2.5$, $\phi_6 = 2.3$, $\phi_7 = 2$, $\phi_8 = 1.4$, $\phi_9 = 1$. Answer based on the following numerical schemes:



(a) Upwind scheme (1 mark)

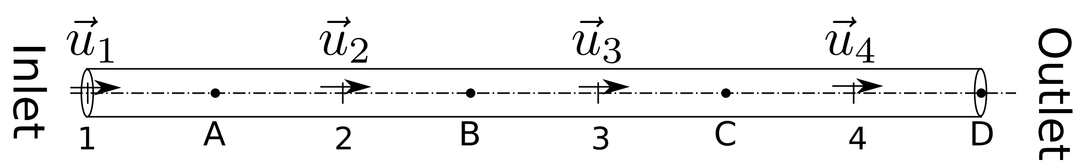
(b) Central scheme (1 mark)

(c) QUICK scheme (1 mark)

(d) TVD scheme with min-mod limiter (2 mark)

(e) TVD scheme with SUPERBEE limiter (2 mark)

6. A liquid of density $\rho = 1$ kg/m^3, viscosity $\mu = 1$ Pa$\cdot$ s flows in a pipe of constant cross-sectional area $A = 1m^2$ shown in the figure below. The pressure is stored at alphabets and velocity is stored at numbers. We want to solve the steady state distribution of velocity and pressure using SIMPLE algorithm. The boundary conditions are: $u_1 = 5$ m/s, $P_D = 0$ Pa and the initial guess for velocity field is $u_2^* = 4$ m/s, $u_3^* = 3$ m/s and $u_4^* = 2m/s$ and for pressure as $P_A = 3$ Pa, $P_B = 2$ Pa, $P_C = 1$ Pa.



Answer the following questions:

(a) Write the discrete velocity correction equation in matrix form. (1 mark)

(b) Calculate the values of the matrix coefficients and the right hand side vector in the above derived equations based on initial guess. (2 marks)

(c) Solve the system for new velocities. (1 mark)

(d) Write the discrete equation for pressure correction in matrix form. (1 mark)

(e) Calculate the values of the matrix coefficients and the right hand side vector in the above derived system. (2 marks)

(f) Solve the above system and calculate pressure correction values. (1 mark)

(g) Calculate new pressures at the end of first outer iteration. (1 mark)

(h) Calculate new velocities at the end of first outer iteration. (1 mark)