Birla Institute of Technology and Science Pilani
ME F485: Numerical Fluid Flow and Heat Transfer

Mid semester exam, Total 25 marks, 10/03/2022, 2:00 PM to 3:30 PM

1. Consider the following equation:

$$x\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial x \partial y} + y\frac{\partial^2\psi}{\partial y^2} = 0.$$

   Sketch the regions of the $x-y$ plane where the equation exhibits (a) Hyperbolic behaviour, (b) Parabolic behaviour, (c) Elliptic behaviour. Draw three separate figures. (6 marks)

2. Write two strengths and two limitations of the finite difference method. (2 marks)

3. Derive the finite difference expression for the first derivative of a variable ϕ at grid point i involving values of ϕ at (i), $(i+1)$, $(i+2)$ and $(i+3)$ grid points. What is the order of accuracy of the expression obtained? (5 marks)

4. The Richardson finite difference scheme for the unsteady heat equation in one dimension $\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}$ consists of central differencing for the temporal term involving values of temperatures at $n-1$ and $n+1$ time levels at grid point j and central differencing in space involving temperature values at $j-1$, j and $j+1$ grid points but at time level n.

   (a) Write the finite difference scheme. (1 mark)

   (b) Do consistency analysis and tell whether the scheme is consistent with PDE. (2 marks)

   (c) What is the order of accuracy of the scheme in space and time? (2 mark)

   (d) Do Von Neumann stability analysis and comment on stability of the scheme. (2 marks)

5. Write the steps involved in the Projection method used for simulating incompressible flows. What are the boundary conditions used for velocity and pressure at inflow, outflow, wall and symmetry type of boundaries. (5 marks)