BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE

First Semester 2022-23

Mid-semester Examination (04th Nov 2022)

Course No. ME G511 **Mechanism and Robotics**

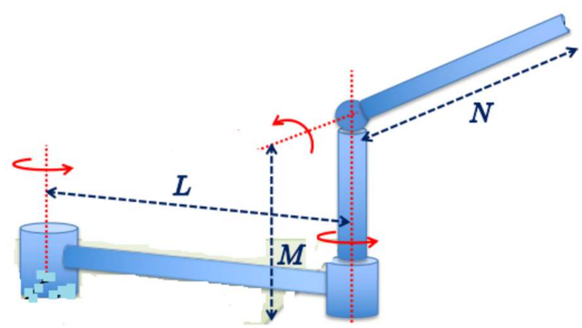
Total Marks: 30 **1 hour 30 Min**

Question 1. Read the statements given below and state "True" or "False" with valid justification [1M each] Answers should not be more that 2/3 lines.

a. Inverse of Homogenous transformation matrix is same as transpose of the same matrix.
b. If the robot is in a singularity configuration, then there is always an infinite number of inverse solutions.
c. A 6-DOF Cartesian robot with a spherical wrist has two inverse solutions when the robot is out of singularity configuration.
d. If a closed-form inverse solution is not known in advance, a numerical method cannot provide one.
e. For a planar manipulator with no of joints $n \geq 3$ (revolute joints) has up to n inverse solutions for a positioning task.
f. Rotation matrix R has a determinant value equal to -1.
g. A 6R spatial robot without a spherical wrist or spherical shoulder has no closed-form inverse solution.
h. A 3-DOF gantry-type robot has only one inverse kinematic solution in its workspace.
i. Freudenstein's equation for four bar mechanism is not scale invariant.
j. Small rotation of a frame about an arbitrary axis is order dependent.
k. All mechatronic devices are robots [Bonus] [10]

Question 2. For the figure given, answer the following

a. Determine the forward Kinematic Model of the arm. Identify the arm with type of joints and DOF of the model.
b. Determine the inverse kinematics model of the arm
c. Determine the jacobian matrix and singularity configuration [5+3+5]



Question 3.

(a) For the close loop mechanism, i.e. four bar mechanism, assign frames using D-H conventions and derive the loop closure equation using the kinematic models. Why the Freudenstein's equation is scale invariant? [3]

(b) A rigid body is rotated first by an angle $\theta=\pi/3$ around the unit vector $r = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$ and then by an angle $\phi=-\pi/3$ around the fixed y-axis. What is the final orientation of the body? [2]

(c) List down the steps required to derive the angular velocities of the coupler and follower for the known angular velocity of crank. [2]

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	Important Formula Set
1	$\mathbf{T}=\begin{bmatrix} R & D \\ O & 1 \end{bmatrix}$
2	${}^{i-1}\mathbf{T}_i=\begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$
3	$\boldsymbol{R}_k(\theta)=\begin{bmatrix} k_x^2 V\theta + C\theta & k_x k_y V\theta - k_z S\theta & k_x k_z V\theta + k_y S\theta \\ k_x k_y V\theta + k_z S\theta & k_y^2 V\theta + C\theta & k_y k_z V\theta - k_x S\theta \\ k_x k_z V\theta - k_y S\theta & k_y k_z V\theta + k_x S\theta & k_z^2 V\theta + C\theta \end{bmatrix}$
4.	$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}=\begin{bmatrix} 1-2\varepsilon_2^2-2\varepsilon_3^2 & 2(\varepsilon_1\varepsilon_2-\varepsilon_3\varepsilon_0) & 2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_0) \\ 2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_0) & 1-2\varepsilon_1^2-2\varepsilon_3^2 & 2(\varepsilon_2\varepsilon_3-\varepsilon_1\varepsilon_0) \\ 2(\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_0) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_0) & 1-2\varepsilon_1^2-2\varepsilon_2^2 \end{bmatrix}$
5,	$\frac{a^2+c^2+d^2-b^2}{2ac}+\frac{d}{a}C_4-\frac{d}{c}C_2=C_2C_4+S_2S_4$
6.	$R_3+R_2\,C_4-R_1\,C_2=\cos(\theta_2-\theta_4)$
7.	$\dot{\mathbf{R}}(t)=\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}\mathbf{R}(t)$
8.	${}^1\mathbf{v}_Q={}^1\mathbf{v}_p+{}^1\mathbf{T}_2\,{}^2\mathbf{v}_Q+{}^1\boldsymbol{\omega}_2\times{}^1\mathbf{T}_2\,{}^2\mathbf{Q}$
9.	$\omega_i=\omega_{i-1}+{}^0\mathbf{R}_{i-1}\,\mathbf{z}_{i-1}\,\dot{\theta}_i$
10.	$\mathbf{v}_{EE}=\mathbf{v}_n=\sum_{j=1}^{n}\frac{\partial({}^0\mathbf{T}_n)}{\partial q_j}\dot{q}_j\,{}^n\mathbf{D}_n$
11.	$=\begin{cases}\begin{bmatrix}\mathbf{P}_{i-1}\\ \mathbf{0}\end{bmatrix} & \text{for a prismatic joint} \\ \begin{bmatrix}\mathbf{P}_{i-1}\times{}^{i-1}\mathbf{P}_n \\ \mathbf{P}_{i-1}\end{bmatrix} & \text{for a rotary joint}\end{cases}$ $\quad J_i=\begin{bmatrix} -{}^0[\mathbf{n}]_{i-1}\,{}^{i-1}(p_y)_n+{}^0[\mathbf{o}]_{i-1}\,{}^{i-1}(p_x)_n \\ {}^0[a]_{i-1}\end{bmatrix}$