BITS Pilani K.K. Birla Goa Campus
Comprehensive Examination
First Semester 2022-2023

ME G511- Mechanisms & Robotics

Date: 29/12/2022 | Total marks: 70
Time: 10:00 AM – 01:00 PM (3 hours) | No. of questions: 6

Note:

- *The exam is open-book. Students are allowed to carry necessary books and a laptop in the exam hall.*
- *All variables, constants, and annotations carry the same meaning mentioned in the class.*
- *If necessary, make reasonable assumptions for solving the problems and state them clearly in the answer sheet.*

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1. (8 marks) An industrial robot uses camera to measure the position and orientation of a cubic workpiece as shown in Figure 1. Find the transformation ${}^{\text{Base}}[T]_{\text{Obj}}$ if positions of four adjacent corners of the workpiece object, and the transformation matrix of the {Base} frame with respect to {Cam} frame are given as:

$$^{\text{Cam}}\mathbf{O}_{\text{obj}} = \begin{bmatrix} 12 \\ 14 \\ -4 \end{bmatrix}, {}^{\text{Cam}}\mathbf{P}_A = \begin{bmatrix} 12.1743 \\ 14.8420 \\ -5.8057 \end{bmatrix}, {}^{\text{Cam}}\mathbf{P}_B = \begin{bmatrix} 10.0076 \\ 14.0737 \\ -4.1580 \end{bmatrix}, {}^{\text{Cam}}\mathbf{P}_C = \begin{bmatrix} 12.0000 \\ 15.8126 \\ -3.1548 \end{bmatrix}, {}^{\text{Cam}}[T]_{\text{Base}} = \begin{bmatrix} 0 & -1 & 0 & 22 \\ 0 & 0 & -1 & 18 \\ 1 & 0 & 0 & -25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

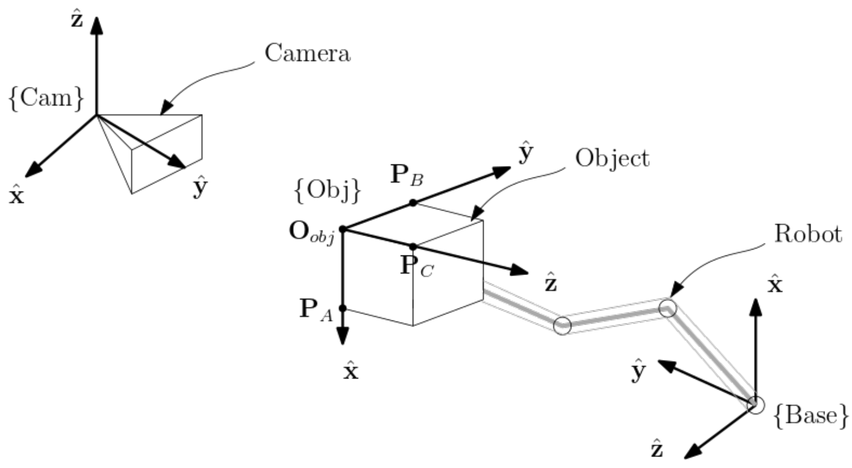


Figure 1: Figure for Qn. 1

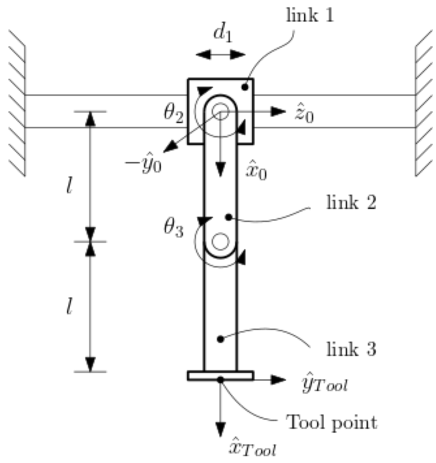


Figure 2: PRR robot for Qn.2

2. The position and orientation of {Tool} frame of the PRR robot shown in Figure 2 can be described by the transformation matrix:

$$^{0}[T]_{\text{Tool}} = \begin{bmatrix} C_{23} & -S_{23} & 0 & lC_{23} + lC_2 \\ 0 & 0 & -1 & 0 \\ S_{23} & C_{23} & 0 & lS_{23} + lS_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $S_{23} = \sin(\theta_2 + \theta_3), C_{23} = \cos(\theta_2 + \theta_3)$, $S_2 = \sin\theta_2, C_2 = \cos\theta_2$.

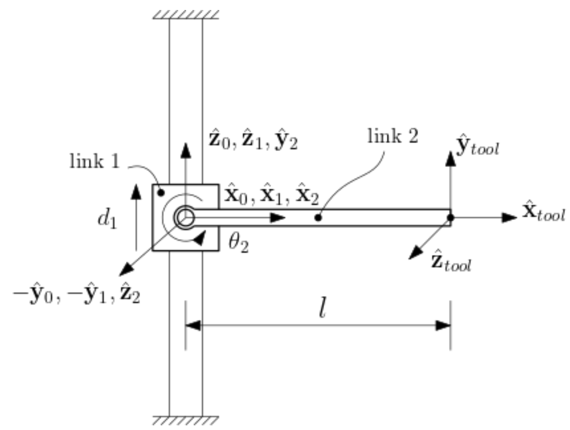
   (a) (12 marks) For $l = 0.4$ m, solve the inverse kinematics (all solutions) and find the values of joint variables d_1, θ_2 and θ_3 (in meters and radians) for the two Tool configurations given below:

$$^{0}[T]^{(1)}_{\text{Tool}} = \begin{bmatrix} 0.3420 & -0.9397 & 0 & 0.4993 \\ 0 & 0 & -1.0000 & 0 \\ 0.9397 & 0.3420 & 0 & 0.7449 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}, {}^{0}[T]^{(2)}_{\text{Tool}} = \begin{bmatrix} 0.9816 & -0.1908 & 0 & 0.7866 \\ 0 & 0 & -1.0000 & 0 \\ 0.1908 & 0.9816 & 0 & -0.0931 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

(P.T.O.)

(b) (6 marks) Design a linear trajectory with parabolic blends for d_1, to move the tool from configuration ${}^0[T]^{(1)}_{\text{Tool}}$ to configuration ${}^0[T]^{(2)}_{\text{Tool}}$ in 5 seconds. Choose $\ddot{q}_c = 6\frac{(q_g - q_s)}{t_g^2}$ (Note: In case of multiple solutions for inverse kinematics, plan the trajectory for any one solution set).

3. (16 marks) Find the dynamic equations of motion of the PR robot shown in Figure 3 *using Lagrangian method.* The first link is a solid block of mass m_1 and the second link is a thin rod with mass m_2. Center of mass of both links are at the center of the links. Force f_1 and torque τ_2 are applied at the joints 1 and 2 respectively.



link (i)	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	0
2	90°	0	0	θ_2
tool	0	l	0	0

Figure 3: PR robot for Qn. 3

4. (12 marks) A 3-RPS manipulator has identical equilateral triangles for top and bottom platforms with $b = 0.17$ m. Direct kinematics analysis for a particular configuration with active joint variables $[l_1, l_2, l_3]^T = [0.2, 0.3, 0.4]^T$ m gives the passive joint variables $[\theta_1, \theta_2, \theta_3]^T = [1.3251, 1.6493, 1.4567]^T$ radians. If the instantaneous joint velocities at this configuration are $[\dot{l}_1, \dot{l}_2, \dot{l}_3]^T = [0.01, 0.01, -0.01]^T$ m/s, what is the *linear velocity* of the centroid of the top platform?

5. (10 marks) Synthesize a 4-bar mechanism which can generate the function $y = 0.5 \tan x$ over the range $0.5 \leq x \leq 1.5$ using three precision points. For the input and output links, choose the starting angles as 30° and 240° respectively. Swing angles of both the links should be 90°. Take the length of fixed link as 0.5 m.

6. A robot is required to perform a grinding task through the following sequence of subtasks (refer Figure 4):

- Subtask 1: The grinding tool is vertically brought down to the surface with velocity v_{descend}
- Subtask 2: The tool is horizontally moved across the workpiece with a velocity v_{grind}, while applying a vertical force f_{grind} on the surface.
- Subtask 3: The tool is vertically raised up with a velocity v_{ascend}

The force required to move the grinding tool horizontally on the surface is very small and can be taken as zero. For each subtask,

(a) (3 marks) Write the natural and artificial constraints

(b) (3 marks) Find the $[S]$ and $[S']$ matrices for hybrid position/force control

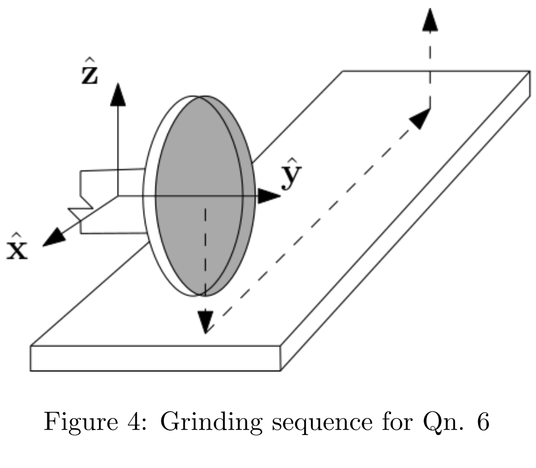


Figure 4: Grinding sequence for Qn. 6

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