Birla Institute of Technology & Science, Pilani
First Semester 2022-2023
Comprehensive Exam

Course No.	:	ME G512
Course Title	:	Finite Element Methods
Nature of Exam	:	Open Book
Weightage	:	35%
Duration	:	3 hours
Date of Exam	:	24/12/2022

No. of Pages = 2
No. of Questions = 5

Note to Students:
1. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
2. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q1. A geometry under plane stress condition is disctretized using a 4-noded triangular element as shown in the **Figure 1** below.

(a) Derive interpolation functions for the 4-noded element in terms of area coordinates (L_1, L_2 and L_3)**.**

(b) Verify that the derived interpolation functions in part (a) are partition of unity shape functions**.**

(c) Use first order approximation of geometry (subparametric formulation) and derive the expression for strain component (ϵ_{xx}) **at node 4** of the triangular element in terms of global coordinates $(x_1, y_1, x_2, y_2, x_3, y_3)$ and displacements $(u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4)$ at the nodes.

[7]

Q2. Consider the 4-noded rectangular element of **Figure 2** with the nodal displacements given by:

$u_1 = 0$ $\quad v_1 = 0$ $\quad u_2 = 0.005$ cm
$v_2 = 0.0025$ cm $\quad u_3 = 0.0025$ cm $\quad v_3 = -0.0025$ cm
$u_4 = 0$ $\quad v_4 = 0$

Assume plane stress condition and use isoparametric formulation to determine the following at **centroidal point P** with $x = 2$ cm, $y = 1$ cm:

(a) In-Plane displacements
(b) In-Plane Strains
(c) In-Plane Stresses

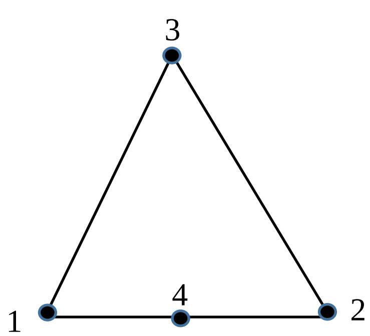

Figure 1

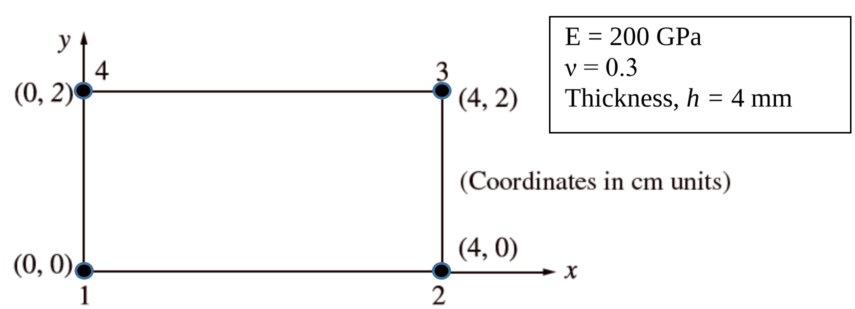

Figure 2

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Q3. The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, k_A = 25 W/m · K and k_C = 60 W/m · K, and known thickness, L_A = 0.40 m and L_C = 0.20 m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B = 0.20 m, but unknown thermal conductivity k_B. Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o}$ = 20°C, an inner surface temperature of $T_{s,i}$ = 600°C, and an oven air temperature of T_∞ = 800°C. The inside convection coefficient h is known to be 25 W/m^2 · K. Use minimum number of 2-noded finite elements and

(a) Determine the value of k_B

(b) Determine the temperature distribution in the wall

[7]

Q4. An isotropic clamped beam (**Figure 3**) of length length 2L with Young's modulus (E), density (ρ), uniform area of cross-section (A_b) and second moment of area (I_b) is supported at center by axial member of same material (E, ρ), length L/2 with cross-section area (A_a). Discretize the beam into two elements of length L and the axial member as a one element of length L/2.

(a) Determine the global stiffness matrix

(b) Determine the global mass matrix

(c) Determine fundamental frequency of vibration of the system.

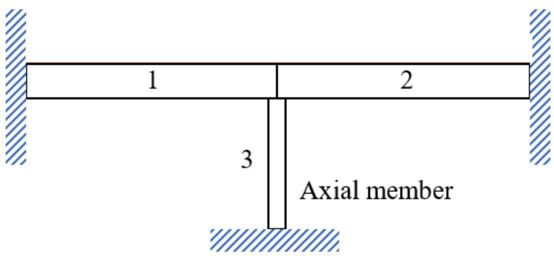

Figure 3

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Q5. Consider the truss structure (L = 1m) as shown in the **Figure 4** below. Use penalty method to impose the boundary conditions and determine displacement (u_s) at **Point 2**.

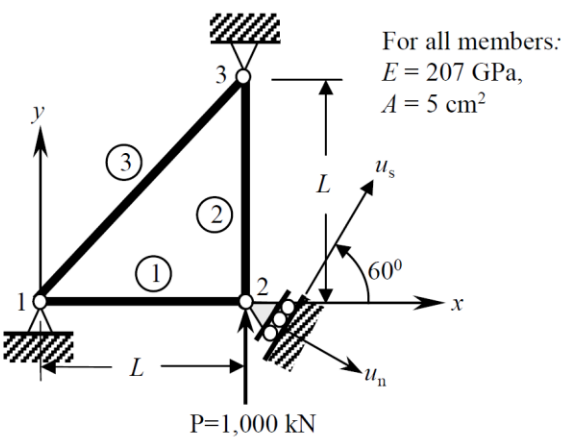

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Figure 4