Birla Institute of Technology & Science, Pilani
First Semester 2023-2024
Comprehensive Exam

Course No.	: ME G512
Course Title	: Finite Element Methods
Nature of Exam	: Open Book
Weightage	: 35%
Duration	: 3 hours
Date of Exam	: 09/12/2023

No. of Pages = 2
No. of Questions = 6

Note to Students:
1. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
2. Assumptions made if any, should be stated clearly at the beginning of your answer.

Q.1. The nodal displacement values (in mm) of the elements shown in **Figure Q1a** and **Figure Q1b** are u_1 = 0.2645, u_2 = 0.2172, u_3 = 0.1800 for the triangular element and u_1 = 0.2173, u_3 = 0.1870, u_2 = u_4 = 0.2232 for the rectangular element. Dimensions of the elements are in meters.

a). Compute u, $\partial u/\partial x$ and $\partial u/\partial y$ at the point (x, y) = (0.375 m, 0.375 m) *for triangular element*.

b). Compute u, $\partial u/\partial x$ and $\partial u/\partial y$ at the point (x, y) = (0.375 m, 0.375 m) *for rectangular element*.

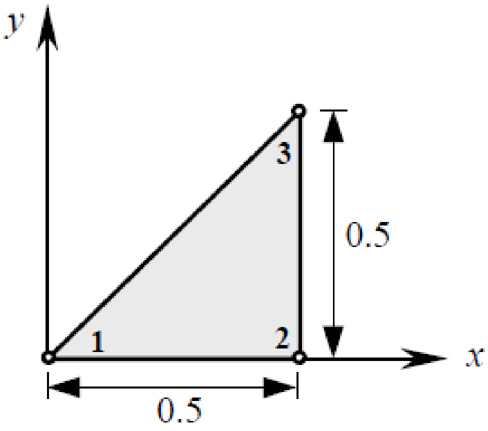

Figure Q1a

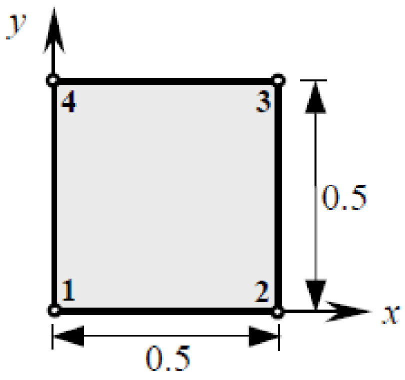

Figure Q1b

[3 + 3 = 6]

Q.2. (a) Determine the interpolation function ψ_9 in terms of area coordinates, L_i, for the triangular element shown in **Figure Q2a.**

(b) Consider the five-noded element shown in **Figure Q2b** and determine interpolation functions for the element in terms of natural coordinates, ξ and η.

[3 + 4 = 7].

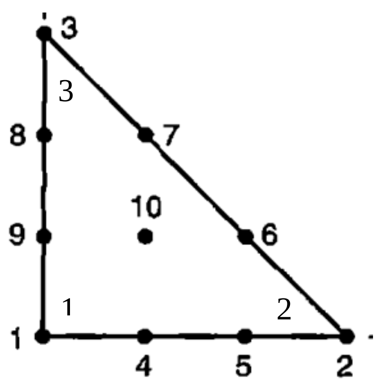

Figure Q2a

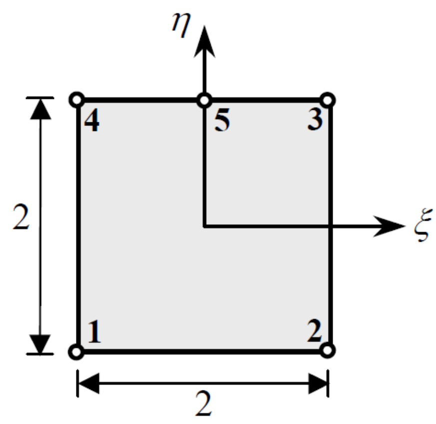

Figure Q2b

Q.3. A bar member ($E = 200$ GPa, $A = 40\ cm^2$) of an aerospace structure is discretised using **one 3-noded bar element, as shown in the Figure 3b below.** Determine the following:

(a) Interpolation functions for the 3-noded bar element shown in **Figure Q3b**.

(b) Global Stiffness Matrix for the bar

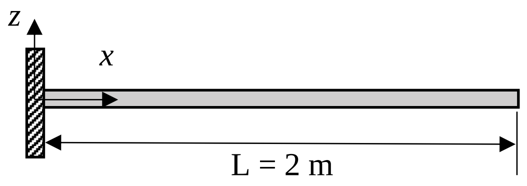

Figure Q3a

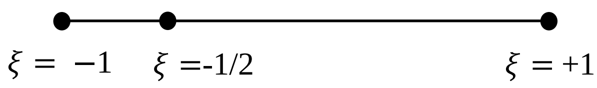

Figure Q3b

[3 + 4 = 7]

Q.4. Determine the following for the beam shown in the **Figure Q4** using **two 2-noded Euler-Bernoulli's beam elements**.

(a) Global Stiffness Matrix

(b) Deflection and Slopes at all nodes

[**3 + 3** = **6**]

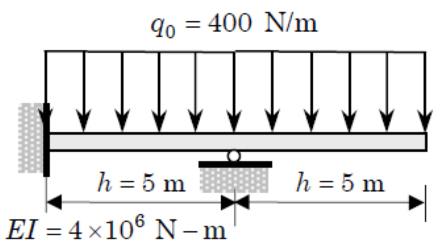

Figure Q4

Q.5. A wall with thickness L, is maintained at temperature T_0 at the left end and is insulated at the right end. The thermal conductivity of the wall material varies linearly as follows:

$$k = k_0\left(1 - \frac{x}{2L}\right) \quad for \quad 0 < x < L, \text{ where } k_0 \text{ is a constant.}$$

Assume one-dimensional heat conduction through the wall, discretise the wall length using only one 2-noded isoparametric element and derive the element temperature matrix (K_{Te}).

[5]

Q.6. The governing equation and boundary conditions for a structural phenomenon are given by:

$$\frac{d^2}{dx^2}\left(k_1\frac{d^2v}{dx^2}\right) + \lambda\frac{d^2v}{dx^2} = 0 \qquad for \quad 0 < x < 1$$

$$v(0) = v(1) = 0, \quad \left(k_1\frac{d^2v}{dx^2}\right)_{\{x=0\}} = \left(k_1\frac{d^2v}{dx^2}\right)_{\{x=1\}} = 0$$

Where k_1 and λ are constants.

Derive the weak form of the governing equation and identify primary and secondary variables

[4]