Birla Institute of Technology and Science Pilani
Department of Mechanical Engineering

# Computational Fluid Dynamics ( ME G515)

Mid sem. Exam., Date: Nov. 01, 2022

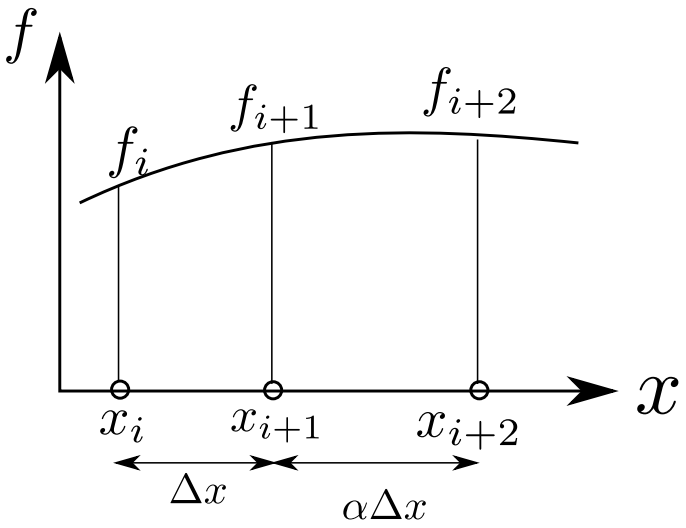
Timing: 11:00 AM to 12:30 PM | First sem. 2022-23 | Max mark: 50

**Write your name and ID at the top clearly. Partial marks will be considered only when the exam. is attempted systematically and neatly.**

1. Classify the following system of equations into hyperbolic, parabolic or elliptic type. (7 marks)

$$\begin{aligned} \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} - \frac{\partial w}{\partial x} &= 0, \\ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{\partial w}{\partial y} &= 0, \\ \frac{\partial v}{\partial y} - \frac{\partial w}{\partial x} &= 0. \end{aligned}$$

2. Using concepts from Taylor series, find the second order accurate forward difference approximation for $\frac{\partial f}{\partial x}|_i$ on an unequally spaced grid as shown in the figure below. (7 marks)




3. Consider the following 1D, steady heat conduction problem

$$\begin{aligned} \frac{d^2T(x)}{dx^2} + \frac{1}{k}g_0 &= 0 \text{ in } 0 < x < L \\ -k\frac{dT(x)}{dx} &= q_0 \text{ in } x = 0 \\ T &= 0 \text{ at } x = L. \end{aligned} \tag{1}$$

Answer the following questions: (4 marks each)

(a) Write the second order accurate finite difference formulation of this problem by dividing the domain $0 < x < L$ into four equal parts.

(b) Calculate the node temperatures for $k = 15$ W/m °$C$, $L = 0.016$ m, $q_0 = 10^6$ W/m$^2$, and $g_0 = 5 \times 10^7$ W/m$^3$.

(c) Derive the expression for the exact solution and calculate the error between the numerical and the exact solution at different grid points.

4. The 'Combined' method for transient, 1D heat conduction equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$ is given by

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \left[ \theta * \frac{T_{i-1}^{n+1} - 2T_i^{n+1} + T_{i+1}^{n+1}}{\Delta x^2} + (1-\theta) * \frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} \right].$$

where the constant $\theta$ is the weight factor that expresses the degree of implicitness. The value of $\theta$ can be between 0 and 1 with $\theta = 0$ giving explicit, $\theta = 0.5$ giving Crank-Nicolson, and $\theta = 1$ giving fully implicit scheme. Answer the following questions:

(a) What value of $\theta$ will given 2nd order accuracy in time and 4th order accuracy in space? (7 marks)

(b) Comment on the stability of the method for $0 \leq \theta \leq 0.5$ and for $0.5 \leq \theta \leq 1$. (7 marks)

5. Briefly describe the need for staggered grids in CFD. Describe how you can solve for unsteady, incompressible flows on such grids. (5 marks)

6. Briefly describe how you can solve for compressible flows using Maccormack scheme. What is the main limitation of Maccormack scheme. (5 marks)