Birla Institute of Technology and Science Pilani
Department of Mechanical Engineering
Computational Fluid Dynamics (ME G515)
Compre. Exam., Date: Dec. 20, 2022

Closed Book	02:00 PM to 3:00 PM	First sem. 2022-23	Max mark: 20

Write your name and ID at the top clearly. Partial marks will be considered only when the explanation is relevant and the exam. is attempted systematically and neatly.

1. Write the title of your project. Who is / are your project partner/s? What is your contribution in the project and your partner's contribution? Which software you used for executing the project? How the mesh was generated? (2 marks)
2. Write two main strengths and two main limitations of Finite Difference Method. (2 marks)
3. Write two main strengths and two main limitations of Finite Volume Method. (2 marks)
4. When you should use explicit numerical schemes and when implicit ones? Give practical examples of physical flow situations for each case. (2 marks)
5. Briefly explain what is order of accuracy of a numerical scheme. Why one should consider the use of higher order methods? Write two negative points of using higher order method. (2 marks)
6. What was the purpose of introducing the Lax-Wendroff scheme and the main problem associated with it? How the Maccormack scheme removed that problem? (2 marks)
7. How can one decide when to stop iterating while solving a system of linear equations using an iterative method? (2 marks)
8. You want to solve a system of non-linear equations where the matrix coefficients are functions of the unknowns. What approach you will use to solve such a system. Which numerical method (direct or iterative) you should use for inner iterations and why? (2 marks)
9. What is the role of under-relaxation factors while solving for incompressible flows with SIMPLE algorithm? (2 marks)
10. What is the main problem if you solve for incompressible flows on co-located grids? What technique is used to alleviate that? How does that technique works? (2 marks)

Birla Institute of Technology and Science Pilani

Department of Mechanical Engineering

Computational Fluid Dynamics (ME G515)

Compre. Exam., Date: Dec. 20, 2022

Open Book | 02:00 PM to 5:00 PM | First sem. 2022-23 | Max mark: 50

Write your name and ID at the top clearly. Partial marks will be considered only when all the sub-steps are clearly shown and the exam. is attempted systematically and neatly.

1. Classify the following system of equations into hyperbolic, parabolic or elliptic type.

$$\begin{aligned} a_1 \frac{\partial u}{\partial x} + a_2 \frac{\partial v}{\partial y} &= g_1, \\ b_1 \frac{\partial v}{\partial x} + b_2 \frac{\partial u}{\partial y} &= g_2. \end{aligned}$$

   Consider three cases where (3 marks each)

   (a) $a_1 = b_1 = a_2 = b_2 = 1$

   (b) $a_1 = b_2 = 1$, $b_1 = 0$, $a_2 = -1$

   (c) $a_1 = b_1 = b_2 = 1$, $a_2 = -1$

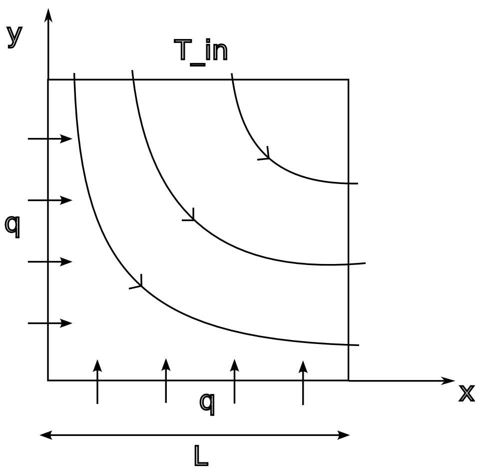
2. Given the function $f(x) = sin(2\pi x)$, determine $\frac{df}{dx}$ at $x = 0.375$ using central and upwind biased finite difference representations of order $O(\Delta x^2)$. Use a step size of $\Delta x = 0.01$. Compare the accuracy of two representations with the analytical solution of the first derivative. (8 marks)

3. Consider the following 1D convection diffusion equation (Pe is the Peclet number)

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} = \frac{1}{Pe} \frac{\partial^2 \phi}{\partial x^2}.$$

   If we use explicit Euler scheme for the temporal part, first order Upwind scheme for convection term and central differencing for the diffusion term, what is the stability criterion? What is this criterion if you use the first order downwind scheme for the convection part? (8 marks)

4. Air with temperature $T_{in} = 15°$ C enters a square shaped domain of side length $L = 1$ m from the top boundary as shown in the figure below. The velocity distribution inside the room is given by $u = x$ and $v = -y$. The left and bottom boundaries have a heat flux of $q = 10$ W/m^2 as shown. The density of air is $\rho = 1.2$ kg/m^3, thermal diffusivity is $\alpha = 2.2 \times 10^{-5}$ m^2/s and thermal conductivity is $k = 0.026$ W/m.K. Using Finite volume method and a mesh of four equally sized control volumes, calculate the steady state temperature distribution inside the domain. (8 marks)



5. A long Uranium rod of radius R=0.12 m is stored in a water pool. The rod temperature distribution is governed by the steady heat conduction equation in cylindrical coordinates:

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) + S = 0.$$

Here $k = 21$ W/m·K is the thermal conductivity of Uranium and $S = 1200$ W/m^3 is the rate of heat generation due to nuclear decay process. The heat from the rod surface is taken by water for which convective heat transfer coefficient $h = 51$ W/m^2·K. The water temperature far from the rod is 20° C. Calculate the temperature distribution along the radial direction of rod by using Finite difference method with four grid points along the radius (three grid cells). (9 marks)

6. The steady, fully developed, incompressible, pressure driven flow in a channel of height $2h$ shown below can be solved by simplifying the Navier–Stokes equation to $\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$. The flow is symmetric about the channel center-line. Using four grid points (three mesh divisions) along the lower half height of the channel, calculate the velocity distribution in the channel using Finite volume method. Take $h = 1$ m, $\mu = 1$ Pa.s and $\frac{dp}{dx} = 1$ N/m^3. (8 marks)