BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI

ME G535 Advanced Engineering Mathematics Midsemester Exam

November 4, 2022

Instructions

- There are eight questions. Answer all the questions.
- Answer the questions in the given order. Start each question on a fresh page.

1. (3 marks) The null space of a 3×4 matrix A is $\lambda \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \lambda \in \mathbb{R}$.

   (a) What is the rank of A and the complete solution of Ax=0?
   (b) What is the exact row reduced echelon form R of A?

2. (3 marks) Under what conditions on b_1 and b_2 (if any) does $Ax = b$ have a solution?

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

   Find two vectors in the null space of A, and the complete solution to $Ax = b$.

3. (3 marks) Consider a set A containing all 2 by 3 matrices. Is the set a vector space? What is the basis for this space?

4. (3 marks) Let u=$(\lambda, 1, 0)$, v $= (1, \lambda, 1)$ and w=$(0, 1, \lambda)$. Find all values of λ which make {u,v,w} a linearly dependent subset of $\mathbb{R}^3$

5. (3 marks) Find E^2, E^9 and E^{-1} if

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

6. (5 marks) The second variation of a functional $J : A \to \mathbb{R}$ at $y_0 \in A$ in the direction h is defined by

$$\delta^2 J(y_0, h) \equiv \frac{d^2}{d\epsilon^2} J(y_0 + \epsilon h)|_{\epsilon=0}.$$

   Find the second variation of the functional

$$J(y) = \int_0^1 (xy'^2 + y \sin y')dx, \; y \in C^2[0,1].$$

7. (5 marks) Let $J(y) = \int_0^1 (3y^2 + 2y' + x)dx + y(0)^2$, where $y \in C^2[0,1], y(0) = 1$. Let $y_0 = x^2$ and $h = x$. Find $\delta J[y_0, h]$

8. (5 marks) Find the shortest (extremal) path from (0,0,0) to (1,1,1) on the surface z=xy.