BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI

ME G535: Advanced Engineering Mathematics

Midsemester Exam, 10^{th} October, 2023.

All questions carry equal weightage. Maximum marks = 40. Duration $1\frac{1}{2}h$.

1. Suppose A has eigen value-eigen vectors pairs (0,u), (3,v), (5,w) $\ni$ u, v, w are linearly independent, give a basis for the null space and a basis for the column space of A. Find a particular solution for $Ax = v + w$.

2. Find a matrix whose eigen values are 1, 2 with corresponding eigen vectors $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

3. Find a unitary matrix U and a triangular matrix T so that $U^{-1}AU = T$, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

4. (a) Find an orthonormal basis for the column space of A.

$$A = \begin{bmatrix} 1 & -6 \\ 3 & 6 \\ 4 & 8 \\ 5 & 0 \\ 7 & 8 \end{bmatrix}$$

(b) Write A=QR, where Q has orthonormal columns and R is right triangular matrix. (c) Find the least-squares solution to Ax=b, if b=(-3,7,1,0,4).

5. Find bases for the four fundamental subspaces of

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}.$$

6. Use elimination to find the triangular factors in A=LU, if

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

7.

$$\mathcal{L}y \equiv -\frac{d^2y}{dx^2}, y \in \{C^2[0, 2\pi], y(0) = y(2\pi)\}$$

Show that $\mathcal{L}$, is a self-adjoint operator. What are the eigen vectors of this operator? Show that they are orthogonal.

8. Let the column vector (a, b, c) represent a quadratic polynomial given by $ax^2 + bx + c$.
(a) Write the 3 by 3 matrix D such that

$$D \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} b \\ 2c \\ 0 \end{bmatrix}.$$

(b) Compute D^3 and interpret the results in terms of derivative.
(c) What are the eigen values and eigen vectors of D?