BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI

ME G535: Advanced Engineering Mathematics

Comprehensive Exam (Part A), 8^{th} December, 2023.

All questions carry equal weightage. Maximum marks = 40.

1. Find the SVD of

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

2. For what numbers a and b, the matrices A and B are positive definite?

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{bmatrix}$$

3. The following equation shows the relation between stress and strain components following summation convention. Find the bulk modulus of the solid in terms of λ and μ.

$$\sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij}$$

$$\implies \sigma_{ij}\delta_{ij} = \lambda\epsilon_{kk}\delta_{ij}\delta_{ij} + 2\mu\epsilon_{ij}\delta_{ij}$$

Bulk Modulus κ is defined as

$$\kappa = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})}$$

4. Find the reciprocal basis (e^1, e^2), for $e_1 = [a, b]$, $e_2 = [c, d]$.

5. Evaluate $\epsilon_{ijk}\epsilon_{jkl}\epsilon_{kli}$

6. Find the shortest route from (0,1/5,0) to (1/3,0,10) on the cylindrical surface $9x^2 + 25y^2 = 1$.

7. If A is a symmetric matrix, the minimum of the quadratic functional $Q(x) = \frac{1}{2}x^T Ax - x^t b$ provides the solution for $Ax = b$. Using the above statement, find a variational principle for the following problem.

$$\frac{d^2y}{dx^2} = f(x) \forall x \in [0, 1]$$

$$y(0) = 0, y'(1) = 2$$

8. The basis vectors e'_i and e_i are related as follows:

$$\begin{bmatrix} e'_1 \\ e'_2 \\ e'_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

If $\vec{A} = 2e_1 + 3e_2 + 4e_3$, find the components of A in e'_i basis.