Birla Institute of Technology and Science Pilani

ME G611: Computer Aided Analysis and Design Second Semester 2022-23

Midsemester Exam
13^{th} March, 2023 Duration: 90 min Maximum Marks: 35

---

1. [10 marks] In the figure Q1, the vertex A is at the origin (0, 0). The coordinates of C and D are (10, 5), (5, 10) respectively. The coordinates of P, Q and R are (20, 0), (20+a,a), (20-a,a) respectively, where a=$\frac{5}{\sqrt{2}}$.
(a) Find the isometric transformation matrix which moves the ΔPQR to ΔP'Q'R' such that R' coincides with D, Q' coincides with C, P' lies *inside* the polygon ABCDE.
(b) Find the isometric transformation matrix which moves the ΔPQR to ΔP*Q*R* such that R* coincides with D, Q* coincides with C, P* lies *outside* the polygon ABCDE. Write down the algorithm first before presenting the calculations.

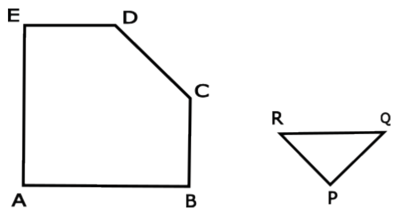

Figure Q1: Isometric Transformation

Solution: Method: It is given Q'→C, R'→D. We need to move PQR to P'CD. From the geometry we can determine the position of P' as (5,5) if inside and (10,10) if outside. Since we have initial and final coordinates and the transformation matrix is given as isometric, we can easily find the matrix using the matrix inverse method.

$$\begin{bmatrix} 5 & 10 & 5 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix} = T_1 \begin{bmatrix} 20 & 20+a & 20-a \\ 0 & a & a \\ 1 & 1 & 1 \end{bmatrix} = T_1 \begin{bmatrix} 20 & 20.5355 & 16.4645 \\ 0 & 3.5355 & 3.5355 \\ 1 & 1 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 5 & 10 & 5 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 20 & 20.5355 & 16.4645 \\ 0 & 3.5355 & 3.5355 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 5 & 10 & 5 \\ 5 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -0.2828 & 1 \\ 0.1414 & 0.1414 & -2.8284 \\ -0.1414 & 0.1414 & 2.8284 \end{bmatrix} = \begin{bmatrix} 0.7071 & 0.7071 & -9.1421 \\ -0.7071 & 0.7071 & 19.1421 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T2 = \begin{bmatrix} 10 & 10 & 5 \\ 10 & 5 & 10 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -0.2828 & 1 \\ 0.1414 & 0.1414 & -2.8284 \\ -0.1414 & 0.1414 & 2.8284 \end{bmatrix} = \begin{bmatrix} 0.7071 & 0.7071 & -4.1421 \\ -0.7071 & 0.7071 & 24.1421 \\ 0 & 0 & 1 \end{bmatrix}$$

2. [10 marks] Determine tangent $\hat{\mathbf{T}}$, normal $\hat{\mathbf{N}}$, binormal $\hat{\mathbf{B}}$ vectors, curvature κ, and torsion τ for the curve $\mathbf{r}(u) = [u^3 - u^2, u^2 + u^3, u + u^2 + u^3]^T$ at $u = 0$. State if the curve is planar or 3 dimensional.

$$\mathbf{r}(u) = \begin{bmatrix} u^3 - u^2 \\ u^2 + u^3 \\ u + u^2 + u^3 \end{bmatrix}, \mathbf{r}'(u) = \begin{bmatrix} 3u^2 - 2u \\ 2u + 3u^2 \\ 1 + 2u + 3u^2 \end{bmatrix}, \mathbf{r}''(u) = \begin{bmatrix} 6u - 2 \\ 2 + 6u \\ 2 + 6u \end{bmatrix}, \mathbf{r}'''(u) = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$\hat{\mathbf{T}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \hat{\mathbf{B}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \hat{\mathbf{N}} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, c = 1, \kappa = 2.8284, \tau = -3$$

Since the $\tau \neq 0$, the curve is not a planar curve.

3. [5 marks] $v = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ is a quaternion. Find the v^{-1}, v^7, v^8.

Since the v is unit quaternion, $v^2 = -1$,
$v^{-1} = -v$,
$v^7 = v^6 v = (-1)^3 v = -v$
$v^8 = (-1)^4 = 1$

4. [5 marks] Consider two frames.
(i) Frame A with origin at (0,0,0) the x-axis aligned in the direction of (-0.6101, 0.5428, 0.5772) the y-axis aligned in the direction of (-0.4169, 0.3996, -0.8164) and the z-axis aligned in the direction of (-0.6738, -0.7387, -0.0175).
(ii) Frame B with origin at (0,0,0) the x-axis aligned in the direction of (-0.4129, 0.8744, 0.2547) the y-axis aligned in the direction of (-0.6783, -0.1086, -0.7267) and the z-axis aligned in the direction of (-0.6077, -0.4729, 0.6380). Given that both the frames are orthogonal matrices, find the rotation matrix which will move frame A to coincide with frame B.

$$A = \begin{bmatrix} -0.6101 & -0.4169 & -0.6738 \\ 0.5428 & 0.3996 & -0.7387 \\ 0.5772 & -0.8164 & -0.0175 \end{bmatrix} B = \begin{bmatrix} -0.4129 & -0.6783 & -0.6077 \\ 0.8744 & -0.1086 & -0.4729 \\ 0.2547 & -0.7267 & 0.6380 \end{bmatrix}$$

To find T_R where

$$B = T_R A$$
$$T_R = BA^{-1} = BA^T$$
$$T_R = \begin{bmatrix} 0.9442 & -0.0463 & 0.3261 \\ -0.1696 & 0.7806 & 0.6016 \\ -0.2823 & -0.6234 & 0.7291 \end{bmatrix}$$

5. [5 marks] Find the orthographic projection of a circle $\mathbf{r}(u) = (2\cos u, 2\sin u, 0)^T$ on the plane x+y+z=10.

Solution:

$$\Pi = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 10 \end{bmatrix} V = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} T_{Proj} = \begin{bmatrix} -2 & 1 & 1 & -10 \\ 1 & -2 & 1 & -10 \\ 1 & 1 & -2 & -10 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\mathbf{r}^*(u) = \begin{bmatrix} \frac{4c-2s+10}{3} & \frac{-2c+4s+10}{3} & \frac{-2c-2s+10}{3} & 1 \end{bmatrix}^T$$

Where $c := \cos u$ and $s := \sin u$