Birla Institute of Technology and Science Pilani
K. K. Birla Goa Campus
Comprehensive Examination 2022-23

Advanced Heat Transfer (ME G631)

Date: 30-12-2022 | Time: 02.00 PM - 05.00 PM | Total Marks: 80

Instruction:

- All questions are compulsory.
- Answer all parts of the question in the same place and start each question in a new page.
- Symbols have their usual meaning.
- Make suitable assumptions whenever necessary. Please state your assumptions clearly.

Q 1. A semi-infinite medium ($0 \leq x \leq \infty, -\infty \leq y \leq \infty, -\infty \leq z \leq \infty$), is initially at a uniform temperature T_i. For time $t > 0$, the boundary surface at $x = 0$ is kept at a constant temperature $T_s > T_i$. The governing equation for 1-D transient heat conduction in the semi-infinite medium is given as

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

(a) Apply scaling analysis to the Eq. (1), and show that the order-of-magnitude of penetration depth δ is given as $\delta \sim \sqrt{\alpha t}$. [2]

(b) Explain the idea of a similarity solution using the temperature profiles for different times. [2]

(c) Solve the Eq. (1) using the similarity method and obtain an expression for the temperature distribution $T(x,t)$. [10]

Q 2. Consider a steady, two-dimensional, incompressible flow with constant properties over a flat plate of length L. The fluid approaches the plate with a free-stream velocity u_∞ and a temperature T_∞. The boundary layer equations with no body force and without heat generation are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Assume $Pr \ll 1$,

(a) Apply scaling analysis to the continuity equation, Eq. (2), and show that $v \sim (\delta_t/L)u_\infty$. Here δ_t denotes the thickness of the thermal boundary layer. [2]

(b) From scaling of the energy equation, Eq. (4), prove that [4]

$$\frac{\delta_t}{L} \sim \frac{1}{\sqrt{Re_L Pr}}$$

(c) Using the definition of the heat transfer coefficient, show that the Nusselt number Nu is given as [4]

$$Nu \sim Re_L^{1/2} Pr^{1/2}$$

Clearly state all the assumptions made.

Q 3. A highly viscous fluid is forced to flow through a straight pipe of inner radius r_0. The effect of friction (viscous shearing) tends to warm up the fluid as it advances through the pipe. This effect is offset by the cooling provided all along the pipe wall, which is isothermal (T_s = constant). The flow is hydrodynamically and thermally fully developed. The energy equation for a fluid with constant properties reduces in this case to

$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \mu\Phi = 0$$

where Φ is the viscous dissipation function and given as

$$\Phi = \left(\frac{du}{dr}\right)^2$$

and the velocity profile is given by

$$u = 2u_m\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

where u_m is the mean velocity.

(a) Determine the temperature distribution through the fluid. [10]

(b) The wall heat flux. [2]

Q 4. Experiments have shown that, for airflow at $T_\infty = 35\,^\circ\mathrm{C}$ and $V_1 = 100\,\mathrm{m/s}$, the rate of heat transfer from a turbine blade of characteristic length $L_1 = 0.15\,\mathrm{m}$ and surface temperature $T_{s,1} = 300\,^\circ\mathrm{C}$ is $q_1 = 1500\,\mathrm{W}$. What would be the heat transfer rate from a second turbine blade of characteristic length $L_2 = 0.3\,\mathrm{m}$ operating at $T_{s,2} = 400\,^\circ\mathrm{C}$ in airflow of $T_\infty = 35\,^\circ\mathrm{C}$ and $V_2 = 50\,\mathrm{m/s}$? The surface area of the blade may be assumed to be directly proportional to its characteristic length. [10]
Properties of air at $T_\infty = 35\,^\circ\mathrm{C}$:

$\rho = 1.16\,\mathrm{kg/m^3}$ $\qquad$ $k = 26.3 \times 10^3\mathrm{W/mK}$

$\nu = 15.89 \times 10^{-6}\mathrm{m^2/s}$ $\qquad$ $\alpha = 22.5 \times 10^{-6}\mathrm{m^2/s}$

$Pr = 0.71$

Q 5. (a) Draw the variation in Nu_x and h_x with x in the case of a laminar flow in a pipe with the surface maintained at a constant temperature. [2]

(b) For flow through a constant cross-sectional pipe, the variation of mean fluid temperature T_m along the flow direction x is given by [8]

$$\frac{dT_m}{dx} = \frac{q''p}{\dot{m}c_p}$$

where $q'', p, \dot{m}$, and c_p denote the heat flux, perimeter, mass flow rate, and specific heat, respectively. The constant heat flux (q'' = constant) condition is maintained at the pipe surface. Obtain the expression for the variation of the mean fluid temperature T_m and the surface temperature T_s in the flow direction. Plot the variation of T_m and T_s with x.

(c) For flow of a liquid metal through a circular tube of radius r_0, the velocity and temperature profiles at a particular axial location may be approximated as being uniform and parabolic, respectively. That is, $u(r) = C_1$ and $T(r) = T_s + C_2[1 - (r/r_0)^2]$, where C_1 and C_2 are constant, and T_s is the surface temperature at that location. What is the value of Nusselt number Nu_D at that location? [8]

Q 6. Discuss different boiling regimes of a pool boiling curve. [8]

Q 7. A 6 m long section of an 8 cm diameter horizontal hot-water pipe passes through a large room whose temperature is 20 °C. If the outer surface temperature of the pipe is 70 °C, determine the rate of heat loss from the pipe by natural convection. [8]
Properties of water at the film temperature $T_f = (20 + 70)/2$ °C:

$$Pr = 0.7241 \qquad k = 0.02699 \text{ W/mK}$$

$$\nu = 1.750 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = (1/T_f) \text{ K}^{-1}$$

The natural convection Nusselt number Nu_D correlation is given by

$$Nu_D = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$$