Birla Institute of Technology and Science Pilani
K. K. Birla Goa Campus
Mid-Semester Examination 2022-23

Advanced Heat Transfer (ME G631)

Date: 04-11-2022 Time: 04.00 PM - 05.30 PM Total Marks: 30

Instruction:

- All questions are compulsory.
- Answer all parts of the question in the same place and start each question in a new page.
- Symbols have their usual meaning.
- Make suitable assumptions whenever necessary. Please state your assumptions clearly.

Q 1. Consider a two-dimensional plate of length L and height H, as shown in Figure 1. The left, right, and bottom boundary temperature is fixed at T_1. Whereas the temperature of the upper boundary varies with x as $T = T_1 + T_m \sin\left(\frac{\pi x}{L}\right)$, where T_m is the amplitude of the sine function. [10]

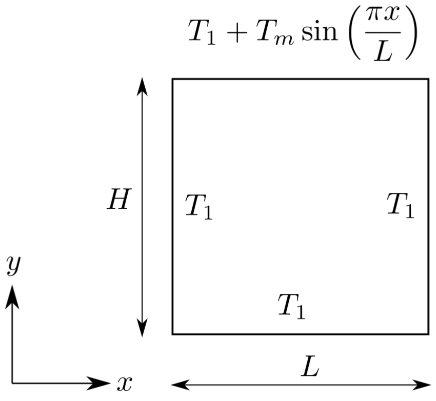

Figure 1: Two-dimensional plate

For two-dimensional steady-state conduction, the governing differential equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Derive an expression for the steady-state temperature distribution $T(x, y)$. If required, use the property of orthogonality of functions as given below.

$$\int_0^L \sin\left(\frac{n\pi}{L}x\right) \sin\left(\frac{m\pi}{L}x\right) dx = \begin{cases} 0 & \text{if } m \neq n \\ L/2 & \text{if } m = n \end{cases}$$

Q 2. Metal plates ($k = 180\,\mathrm{W/m \cdot K}$, $\rho = 2800\,\mathrm{kg/m^3}$, and $c_p = 880\,\mathrm{J/kg \cdot K}$) with a thickness of 2 cm exiting an oven are conveyed through a 10 m-long cooling chamber at a speed of 4 cm/s (Figure 2). The plates enter the cooling chamber at an initial temperature of 700 °C. The air temperature in the cooling chamber is 15 °C , and the plates are cooled with blowing air. The convection heat transfer coefficient is given as a function of the air velocity $h = 33V^{0.8}$, where h is in $\mathrm{W/m^2 \cdot K}$ and V is in m/s. To prevent any incident of thermal burn, it is necessary to design the cooling process such that the plates exit the cooling chamber at a relatively safe temperature of 50 °C or less. Determine the air velocity and the heat transfer coefficient such that the temperature of the plates exiting the cooling chamber is at 50 °C. [6]

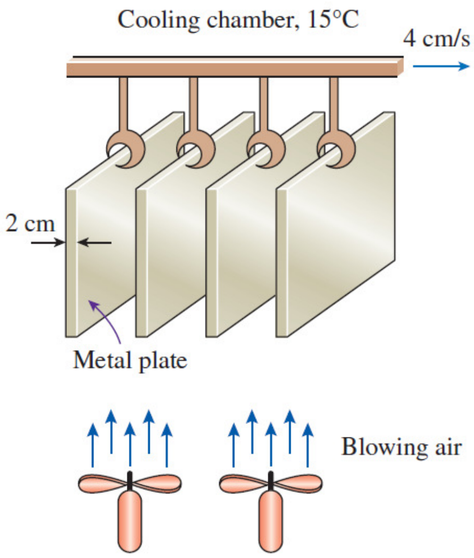

Figure 2: Question 2

Q 3. (a) Consider 1-D, steady-state heat conduction equation in a plate with constant thermal conductivity k in a region $0 \leq x \leq L$. Energy is generated in the medium at a rate of $q_0 e^{-\beta x}$ (W/m^3), while the boundary surfaces at $x = 0$ are kept insulated and $x = L$ dissipate heat by convection into a medium at temperature T_∞ with a heat transfer coefficient h. Give the heat conduction equation and boundary equations for this problem. [2]

(b) The two dimensionless numbers describe the transient conduction: the Biot number (Bi) and the Fourier number (Fo). Explain the use of the lumped capacitance model, exact solution, and semi-infinite solid approximation using these dimensionless numbers. [2]

(c) What is a semi-infinite medium? Give examples of solid bodies that can be treated as semi-infinite mediums for heat transfer purposes. [2]

(d) Derive the following expression for the heat transfer coefficient. [2]

$$h = \frac{-k_f}{(T_s - T_\infty)} \frac{\partial T}{\partial y}\bigg|_{y=0}$$

(e) Use the result of part (a) and show that the Nusselt number Nu is the dimensionless temperature gradient. [2]

(f) In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as [2]

$$\frac{T(y) - T_s}{T_\infty - T_s} = 1 - \exp\left(- Pr \frac{u_\infty y}{\nu} \right) \tag{1}$$

where y is the distance normal to the surface and the Prandtl number, $Pr = 0.7$. If $T_\infty = 400\,\mathrm{K}$, $T_s = 300\,\mathrm{K}$, $k_{air} = 0.0263\,\mathrm{W/m^2K}$, and $u_\infty/\nu = 5000\ \mathrm{m^{-1}}$, what is the surface heat flux?

(g) Derive an expression for the rate of change in temperature recorded by a probe, if the probe is following the fluid motion. [2]